Quality is in the Eye of the Beholder: Taste Projection in Markets with Observational Learning*

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April 25, 2024

Abstract

We study how misperceptions of others' tastes influence beliefs, demand, and prices in markets with observational learning. Consumers infer a good's quality from the quantity demanded and price paid by others. When consumers exaggerate the similarity between their and others' tastes, such "taste projection" generates discrepant quality perceptions, which are decreasing in a projector's taste and increasing in the observed price. These biased inferences produce an excessively elastic market demand. We also analyze dynamic monopoly pricing with short-lived taste-projecting consumers. Optimal pricing follows a declining path: a high initial price inflates future buyers' perceptions and lower subsequent prices induce over-adoption.

JEL Classification: D42; D82; D83; D91.
Keywords: Social Learning; Dynamic Pricing; Projection Bias; False-Consensus Effect.

*For helpful comments, we thank Yair Antler, Benjamin Bushong, Rahul Deb, Erik Eyster, Simone Galperti, Alessandro Lizzeri, Nima Haghpanah, Matthew Rabin, Larry Samuelson, Joshua Schwartzstein, Sevgi Yuksel, the referees, and the audiences at the Australasian Local Economic Theory Seminar (ALETS), the 2021 Australasian Economic Theory Workshop at the University of Sydney, UTS, UCSB, Purdue, University of Queensland, Florida State, Boston College, Università degli Studi di Napoli Federico II, Manchester, Adelaide, Monash, Deakin, Iowa, Vanderbilt, the 2021 Melbourne IO Workshop, the 2021 Asia Pacific Industrial Organization Conference in Singapore, the 2022 IIOC in Boston, the 2022 CSEF-IGIER Symposium on Economics and Institutions in Anacapri, the Workshop on Decision Making under Uncertainty at St. Leo's College (UQ), CEPR Virtual IO Seminar Series, the 2022 Stony Brook International Conference on Game Theory, SAET 2022 in Canberra, and the 2022 Latin American Workshop in Economic Theory.

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1 Introduction

We often use the popularity of a product to assess its quality. We may naturally expect, for instance, that a new electric car has better performance when more people buy it, that a new health trend provides greater benefits when more of our friends adopt it, or that an investment has a higher expected return when our colleagues flock to it. Indeed, a large theoretical and empirical literature has emphasized how observational learning shapes the adoption of new products, spanning consumer goods, entertainment, insurance plans, agricultural technologies, and financial products (see, e.g., Mobius and Rosenblat, 2014 and Bikhchandani et al., 2021 for reviews).

But how does social learning operate when people don't fully appreciate how others' preferences differ from their own? In all the examples above, choices are not driven purely by perceptions of a commonly-valued quality; they also depend on idiosyncratic tastes and motives. For instance, some consumers driving electric vehicles might have a distinct desire to reduce their carbon footprints, and some people investing in cryptocurrencies might be more risk tolerant than others. Yet, do consumers and investors properly account for the fact that others' choices reflect their tastes and not just their private information? Long-standing literatures in psychology on social projection and the false-consensus effect, along with mounting evidence from economics, suggest the answer is no. In particular, people often exaggerate the degree to which others' tastes are similar to their own (Ross et al., 1977; Marks and Miller, 1987; Krueger and Clement, 1994; Engelmann and Strobel, 2012). For example, those with specific tastes for certain consumer products tend to overestimate how many share these tastes (Orhun and Urminsky, 2013). Such misperceptions also arise when evaluating others' risk preferences (Faro and Rottenstreich, 2006), political preferences (Delavande and Manski, 2012), and taste for effort (Bushong and Gagnon-Bartsch, 2023). Moreover, a recent meta-analysis by Bursztyn and Yang (2022) shows that misperceptions of others are widespread in the field, underscoring the importance of further understanding their market implications.

In this paper, we study how such "taste projection" distorts consumers' beliefs, market demand, and prices in a social-learning environment where valuations for a product have both a common and private component. The common component—the product's intrinsic quality—is initially unknown to (some) consumers, who try to infer it from the quantity demanded by others at a given price. Consumers know the private component of their valuation—their idiosyncratic taste for the product—but wrongly "project" this onto others: they exaggerate how similar others' tastes are to their own. We show that taste projection leads consumers to systematically mislearn a product's quality, characterizing how these biased beliefs depend on an individual's own taste and the product's price, and how they ultimately shape market demand. Furthermore, we analyze the optimal pricing strategy of a seller who is aware of consumers' projection.

Our implications are particularly relevant for markets where heterogeneous consumers actively rely on others' choices to guide their own—e.g., those with prominent best-seller lists or a tendency to trend on social media. Consider, for instance, the health and wellness industry, where new products, whose quality is ex-ante uncertain and difficult to ascertain, are routinely introduced; e.g., “innovative” fitness equipment and classes, or “revolutionary” dietary regimens.[1] Consumers’ willingness to pay for such products is of course influenced by their perceived health benefits; yet, consumers likely differ in their idiosyncratic tastes for exercise or a particular diet.

For a concrete example, consider Inês and Peter who are independently contemplating whether to enroll in a fitness program touting some of these new features. Inês enjoys an active lifestyle; Peter does not, but his physician has encouraged him to get in shape. Suppose they both see an article reporting the number of people who joined the program in the past six months. Projection will lead them to draw different inferences about the program’s potential benefits. Projecting her love of fitness onto others, Inês will find the take-up rate disappointingly low. Conversely, the number of adopters will look high to Peter. Hence, they draw conflicting conclusions despite observing exactly the same information—inferred quality is “in the eye of the beholder.” In particular, Inês, who *likes* exercise, forms a more pessimistic inference. Taste projection therefore induces consumers with a stronger idiosyncratic taste for a product to inadvertently be more critical when judging its quality from its popularity. By contrast, Peter becomes too eager to join the program, exaggerating its benefits and potentially over-consuming in various ways (e.g., enrolling in unnecessary classes).

Moreover, because Inês and Peter’s inferences are negatively related to their idiosyncratic taste, their (perceived) total valuations for the program will be too similar. Although the difference between these valuations should be driven solely by the difference in their private values, Inês’s pessimistic inference deflates her total perceived valuation, whereas the opposite holds for Peter. Hence, taste projection is self-fulfilling: because buyers believe that idiosyncratic tastes are less dispersed than they actually are, they will draw divergent inferences about the common value in a way that results in perceived total valuations that are indeed less dispersed.

While the direction of Inês’s and Peter’s misinference will depend on their specific tastes, a more subtle implication of taste projection is that they will both form inferences that increase in the program’s price, irrespective of their taste. Indeed, because projectors underestimate the heterogeneity in others’ valuations, they believe market demand is more elastic than it really is. Therefore, although they correctly predict the take-up rate to decrease with the program’s price, they expect to see even fewer patrons than what a rational consumer would predict as the price increases. To rationalize this higher-than-expected demand, they will conclude that quality is higher when the price is higher. More broadly, projectors systematically overestimate the quality of a product when they see others buying it at a price they themselves are initially unwilling to pay since they over-attribute others’ purchases to positive information rather than differences in tastes. Hence, projection provides a

[1] For instance, the New York Times has noted a growing demand for high-end fitness clubs (“Think Getting Into College Is Hard? Try Applying for These Gyms”: www.nytimes.com/2023/03/25/style/exclusive-gym-memberships.html).

simple yet novel explanation for why quality perceptions are often swayed by prices.[2]

The properties of misinference described above create new incentives for a seller that would not arise under rational learning. First, since perceived quality increases in the observed price, there is a "belief-manipulation effect": in a dynamic setting, a monopolist will set high prices early on to inflate future consumers' beliefs about the value of its product. This holds even when consumers think the seller does not have an informational advantage, and hence it is not driven by classical signaling motives. Second, projectors' perceived valuations being excessively similar creates an "elasticity effect": projectors' demand is more elastic than that of rational agents, and thus a reduction in the current price attracts an even larger share of new consumers. Together, these effects imply that a monopolist's optimal pricing strategy follows a declining path. The seller uses high prices in earlier periods to inflate later consumers' quality perceptions (i.e., creating "hype"), and then reaps the benefits of such manipulation by gradually lowering the price to induce over-adoption.[3]

We present our model in Section 2. We focus on a setting with a continuum of consumers deciding whether to buy a product with an uncertain quality, $\omega \in \mathbb{R}$. Each consumer i's valuation is increasing in both ω and their private value, or "taste," t_i. Some consumers observe a signal s correlated with ω while others are uninformed and rely on social learning to estimate ω. In our setting, uninformed but rational agents would correctly infer the signal s from the market demand given the price. This provides a simple environment to study the effects of taste projection, since any learning failures arise from projection itself rather than other frictions to information aggregation.

Our model of taste projection applies Gagnon-Bartsch et al.'s (2021a) framework to our setting. Individuals hold misspecified models about the distribution of tastes: private values are in fact independently drawn from a distribution F, yet an individual with private value t_i mistakes F for a distribution $\widehat{F}(\cdot|t_i)$ that is overly concentrated around his own value. We close the model by assuming individuals are naive about their own bias and that of others, but are otherwise rational. Hence, each individual i thinks everyone agrees that private values are distributed according to $\widehat{F}(\cdot|t_i)$.

We begin our analysis in Section 3 by studying a simple static model with a fixed price, reflecting the logic of a rational-expectations equilibrium, albeit with misspecified expectations. As foreshadowed in our example, we show that taste projection has three main effects. First, a consumer's perceived quality is negatively related to his taste: when his private value is higher, he expects the good to be more attractive to others, which makes him more pessimistic about its quality. We further discuss how this prediction is supported by recent experimental evidence (Gagnon-Bartsch and

[2] A large literature, primarily in marketing, has documented a positive relationship between prices and perceived quality; see Rao and Monroe (1989) and Völckner and Hofmann (2007) for reviews. This perceived relationship emerges even when the true relationship is weak or non-existent (e.g., Gerstner, 1985 and Broniarczyk and Alba, 1994) and is strengthened in settings where, as in our model, people observe others' purchase decisions (Yan and Sengupta, 2011).

[3] In this way, we provide a novel explanation for why advertising high previous prices can persuade consumers to buy at a new lower price. This contrasts with other explanations based on salience (e.g,. Bordalo et al., 2013, 2020) or an intrinsic "taste for bargains" (e.g., Jahedi, 2011; Armstrong and Chen, 2020), and it arises even when prices do not rationally signal quality (as in, e.g., Bagwell and Riordan, 1991 or Taylor, 1999).

Bushong, 2023). Second, each buyer's perceived quality is increasing in the price. Indeed, because he underestimates the heterogeneity in tastes, a projector's conjectured demand curve is a counter-clockwise rotation of the true one. Thus, if the price were to increase, then the quantity demanded would fall by less than what a projector would predict under the beliefs he formed at the original price. The projector's perceived quality consequently increases to compensate for this less-than-predicted drop in quantity demanded. Third, perceived total valuations are less dispersed than under rational learning. Although a projector with a high private value perceives a greater benefit from adoption than one with a low value, the wedge between these perceptions is diminished relative to the rational benchmark. This results in a volatile market demand that is excessively elastic.

In Section 4 we analyze a dynamic version of our model. Our first result highlights how the demand of taste-projecting consumers overreacts to price changes: a price cut attracts too many consumers by moving the marginal buyer into the region of types who overestimate quality, whereas a price hike excludes too many for the opposite reason. In turn, this introduces an intertemporal link in the seller's pricing incentives. In our simple environment, the seller's optimal strategy under rational learning is to continually charge the static monopoly price. With projection, however, the seller prefers a decreasing price path. Since demand overreacts to price changes, undercutting the previous price attracts too many consumers in the current period. Yet, increasing the price boosts the perceived quality of future consumers at the cost of forgoing current sales. The seller optimally balances these effects by setting an inflated initial price above the static monopoly price and then reducing it.[4] We also show that projection increases the seller's profit and discuss its effect on welfare. While the expansion of sales in later periods can shrink the traditional monopoly deadweight loss, projection can introduce new forms of inefficiency. As low types tend to overestimate quality, they are systematically lured into buying even when they should not. Thus, the seller's manipulative pricing induces excessive take-up among uninformed buyers, consistent with notions of herding or bandwagon effects. In fact, when projection is sufficiently strong, even consumers valuing the good below its cost are lured into buying. While our dynamic analysis mostly focuses on the tractable two-period case, we also describe how many of our results generalize to longer horizons.

Section 5 analyzes two extensions of our model. First, we consider a two-period setting with "long-lived" consumers who can choose when to buy, and we show that projectors still over-adopt the good even when the price is fixed. A selection effect naturally emerges: high types buy in the first period whereas uninformed low types delay in order to glean information from early adopters. Projectors who delay under-appreciate this selection effect, since they underestimate the taste difference between early adopters and themselves. Thus, they tend to overestimate quality, which

[4]This manipulating role of high initial prices is reminiscent of other strategies discussed in the literature. Stock and Balachander (2005) show that a monopolist might make a product scarce in order to signal its quality; similarly, Miklós-Thal and Zhang (2013) argue that "demarketing" strategies that discourage consumers (e.g., limited advertising, understocking inventory) can raise the product's perceived quality. Compared to this literature, we emphasize a different mechanism through which restraining initial sales via high prices can inflate later consumers' quality perceptions.

generates excessive demand and systematic disappointment among these consumers.[5] Second, we revisit the static equilibrium from Section 3 but allow for multi-unit demand, and we show that projectors with a high private value will under-consume while those with a low one will over-consume. Thus, all projectors experience inefficiencies, and those with more extreme tastes suffer more.

Section 6 concludes by discussing the robustness of our analysis as well as some additional applications of our framework. In particular, we describe how our results continue to hold for richer information and valuation structures, and in the presence of competition between multiple sellers. Moreover, we point out some further implications of taste projection for price discrimination and for how individuals ascertain the reliability of different information sources.

Related Literature

We contribute to a recent literature exploring how specific behavioral biases, along with more general forms of model misspecification, interfere with social learning. Much of this literature considers environments similar to the sequential "herding" models of Banerjee (1992), Bikhchandani et al. (1992), and Smith and Sørensen (2000) and studies when beliefs may converge on a false state of the world or fail to converge at all. For instance, Eyster and Rabin (2010), Bohren (2016), and Gagnon-Bartsch and Rabin (2021) examine how neglecting the redundancy of information in others' actions can lead society to grow convinced of a false state. Guarino and Jehiel (2013) study agents who infer from aggregate statistics rather than the precise history. Bohren and Hauser (2021) and Frick et al. (2023) provide frameworks for studying the convergence of beliefs under a wide range of misspecified models. Closer to the specific error we study, Frick et al. (2020) show that when agents share a common misperception of the type distribution, even small amounts of misspecification can cause incorrect learning. Gagnon-Bartsch (2016) considers a simple variant of taste projection with two types who hold conflicting misperceptions, showing how it can lead to long-run disagreement or beliefs that perpetually cycle. Unlike the papers above, we focus on market outcomes when prices directly influence biased agents' beliefs and examine how a seller would optimally use prices to distort those beliefs.

We also contribute to a literature in IO on pricing in the presence of observational learning. This literature primarily studies rational inference under frictions to information aggregation, analyzing how a seller's behavior either alleviates or intensifies these frictions. Bose et al. (2006, 2008) consider a pure common-value environment with a monopolist who, in each period, faces an uninformed short-lived buyer. Buyers learn about the common value from the history of prices and purchase decisions. Information aggregates slowly because there is a single buyer in each period,

[5] Empirical studies show that second-wave consumers tend to display greater dissatisfaction and suggest that this may stem from consumers neglecting selection (e.g., Li and Hitt, 2008; Dai et al., 2018). Our model provides a specific mechanism explaining why consumers may under-appreciate these selection effects.

and the seller maximizes profits by setting prices that reveal as much information as possible. In a similar setting, Parakhonyak and Vikander (2023) show that a monopolist may optimally create product scarcity in order to trigger a "buying herd." More similar to our setup, Caminal and Vives (1996, 1999) consider a model with a continuum of short-lived consumers who are privately but imperfectly informed about the quality of two competing products. Consumers in a later generation don't observe past prices, but try to infer a product's quality from its market share in the previous period. Differently from us, because consumers cannot see the previous price, sellers set low introductory prices to boost sales in an attempt to convince buyers that their quality is high.

The IO literature above mainly focuses on sellers who do not have an informational advantage over buyers, and we follow in this tradition. However, another related strand of this literature examines how a privately informed seller can signal its quality through prices and other means. While we do not analyze such signaling, some of our predictions resemble those from this literature. For instance, Bagwell and Riordan (1991) analyze a monopolist facing a mix of informed and uninformed consumers (like us), and show that high and declining prices can signal higher quality when the high-quality seller has a sufficiently high cost. In contrast, our mechanism generates quality perceptions that are increasing in price even when a seller's quality is not tightly linked to their costs.[6] Taylor (1999) considers a two-period model with private and common values where a privately-informed house seller faces short-lived buyers who try to infer the house's quality from its time on the market. The optimal price path is declining since a higher initial price sends a less negative signal when the house is not sold. More broadly, we differ from both strands above by considering a setting that neutralizes informational frictions that impede rational learning (e.g., incomplete learning, search costs, or signaling motives) in order to isolate how taste projection itself interferes with learning.

There is also a small but growing theoretical literature on the implications of projection and the false-consensus effect in various domains. Goeree and Grosser (2007) examine how this bias can lead to inefficient election outcomes, while Frick et al. (2022) show how it can arise when agents neglect assortativity in matching. Gagnon-Bartsch et al. (2021a) show how the projection of private values can lead to overbidding and inefficiencies in auctions. Madarász (2012, 2021) and Madarász et al. (2023) study the implications of "information projection"—whereby agents exaggerate the extent to which others know their private information—in several contexts (e.g., communication and bargaining). While obviously related, as private tastes represent idiosyncratic information, their model differs from ours on two key dimensions. First, while in our model a projector thinks that he is more informed about others' tastes than he really is, an information-projecting agent thinks that *others* are more informed than they really are. Moreover, projectors are fully naive in our model, whereas in their model players partially anticipate and best respond to each other's projection.

[6] The optimal price path with signaling can also be increasing if consumers learn about quality or their idiosyncratic tastes from repeat purchases, as in Milgrom and Roberts (1986) and Judd and Riordan (1994). In such cases, the seller may use introductory offers to induce learning and repeat purchases. We focus on a setting without repeat purchases.

2 Model

In this section, we introduce the basic features of the environment and present our model of taste projection. Subsequent sections examine projection in various settings: Section 3 considers inference in a static "steady-state" model, whereas Section 4 considers a dynamic one. We describe the specific features of those settings in their respective sections but introduce their common core here.

2.1 Environment

Types, Payoffs, and Actions. Agents attempt to learn the fixed and commonly-valued quality of a good, denoted by $\omega \in \mathbb{R}$, based on others' purchase decisions. Each individual i's total valuation for the good, $u(\omega, t_i)$, derives from both the common value, ω, and a private value (or "taste") denoted by $t_i \in \mathcal{T} \equiv [\underline{t}, \bar{t}] \subseteq \mathbb{R}$. Private values are i.i.d. across individuals with a CDF $F : \mathcal{T} \to [0, 1]$. We assume that F admits a smooth, positive log-concave density $f \equiv F'$. In our formulation of taste projection detailed below, we assume each agent has a misspecified model of F, treating it as excessively concentrated around his own taste relative to the true distribution. We allow $\mathcal{T}$ to include values such that some types may have a negative valuation for the good; this lets us show how projection may lead to inefficient adoption.

There is a continuum of agents, each of whom makes a once-and-for-all decision about whether to buy a single unit of the good. For simplicity, we assume individual i's total valuation is $u(\omega, t_i) = \omega + t_i$; buying the good at price p yields a payoff of $\omega + t_i - p$, while rejecting it yields a payoff normalized to zero. Each agent's choice maximizes their expected utility given their subjective beliefs over ω. Our static model (Section 3) considers a unit mass of agents, while our dynamic model (Section 4) features a new unit mass of agents entering in each period.

Information Structure. Agents begin with a non-degenerate common prior over ω. We focus on a simple signal structure to make the effects of projection transparent: there is a single signal in the market and a fraction of agents observe its realization, $s \in \mathbb{R}$. Let $\bar{\omega}(s) \equiv \mathbb{E}[\omega|s]$ denote the rational expectation of ω conditional on s. The signal is drawn from a continuous CDF $G(\cdot|\omega)$ that obeys the (strict) Monotone Likelihood Ratio Property in ω so that $\bar{\omega}(s)$ is strictly increasing in s. Informed agents will thus take actions based on $\bar{\omega}(s)$ and uninformed agents try to infer $\bar{\omega}(s)$ from these actions. We assume that $\bar{\omega}(s)$ has full support on $\mathbb{R}$; this simplifies the analysis by guaranteeing that projectors will draw a coherent inference from any possible market outcome (i.e., they never observe outcomes they deem impossible). This signal structure is consistent, for instance, with the familiar Gaussian model where the signal and prior are both normally distributed.[7] Moreover, while this simple structure is sufficient to study several features of misinference arising from taste projection,

[7] We assume individuals have correct perceptions of the signal structure in order to isolate the effects of taste projection from other biases. Differently from Madarász (2012, 2021), individuals in our model do not project their information about ω; however, taste projection will endogenously distort an individual's perception of such information.

the online appendix shows that the main effects of projection on beliefs continue to emerge in richer structures with heterogeneous signals.

Social Learning. Informed consumers decide whether to purchase the good based solely on their signal, and thus on $\bar{\omega}(s)$. Uninformed consumers instead attempt to infer $\bar{\omega}(s)$ from the purchase decisions of others conditional on the price, p. Let $d \in [0,1]$ denote the measure of consumers who buy; i.e., the quantity demanded. In our static model, uniformed consumers infer $\bar{\omega}(s)$ from the market outcome (p, d). Similarly, in our dynamic model, consumers in a given period observe the the market outcome from the previous period. Since we assume $\bar{\omega}(s)$ has full support on $\mathbb{R}$, any pair (p, d) is uniquely rationalized by a feasible value of $\bar{\omega}(s)$ whenever $d \in (0,1)$; the value that rationalizes the data, however, will differ for projectors. Furthermore, since there is a continuum of agents, the behavior of the informed consumers fully reveals $\bar{\omega}(s)$ to the uninformed ones.[8] Correct social learning is therefore immediate in the rational benchmark of our setup. Taste-projecting agents will nevertheless mislearn: although they think $\bar{\omega}(s)$ is fully revealed, their erroneous beliefs about others' tastes lead them to infer an incorrect value.

Prices. We consider two cases regarding the origin of prices. First, we consider exogenously determined prices (e.g., a price-taking seller) and describe beliefs as a function of those fixed prices. Second, we consider a profit-maximizing monopolist. Throughout, we assume the monopolist has a constant marginal cost normalized to zero and, importantly, is aware of consumers' projection bias.

The seller also observes s but does not have any additional private information about ω. Because rational uninformed agents in our setting can fully extract s from others' actions, this assumption guarantees that the seller and rational agents effectively have symmetric information. This neutralizes classical motives for the seller to use prices as signals about ω, which allows us to isolate pricing effects that arise entirely due to taste projection.[9] As such, our focus is on agents drawing inference from demand rather than prices per se. In particular, uninformed agents ask themselves what signal s best explains the quantity demanded d when the price is p, but do not draw any inference about s from the seller's particular choice of price. While this assumption is admittedly strong, it helps simplify and focus our analysis.[10] Yet, this assumption does not imply that consumers ignore

[8] This environment shares similarities with models of sequential observational learning with common preferences in which a single agent acts in each period and takes a continuous action (e.g., Lee, 1993; Eyster and Rabin, 2010). In the rational equilibrium of these models, an agent can perfectly deduce a predecessors' beliefs based on their action. In our setup, an individual agent's action does not reveal their information in the rational equilibrium, but the *aggregate* behavior among the continuum of observed agents will reveal their collective information.

[9] As we discuss further below, uninformed agents who do not directly observe s think they can fully extract s form the market outcome, regardless of the seller's chosen price. Thus, although the seller and some buyers might have asymmetric information ex ante, buyers expect symmetric information at the interim stage (i.e., when making their choices). This expectation is correct for rational buyers. Projecting buyers who misinfer s still think, albeit wrongly, that they share common information with the seller at the interim stage.

[10] We are not unique in this approach. As discussed above, most existing papers on pricing in markets with observational learning either abstract from cases in which the seller uses prices to signal private information or impose other simplifying assumptions.

prices when drawing inference. Indeed, the price is essential for *interpreting* observed demand—conditional on s, observers rightfully expect fewer sales when p is higher. Put differently, agents in our model infer from others' *reaction* to prices, rather than the chosen price itself. The environment we consider is also conducive to this assumption since agents believe that d alone is sufficient to reveal s once they know p, regardless of why p was chosen.[11]

2.2 Taste Projection

Gagnon-Bartsch et al. (2021a) provide a general model of taste projection that is applicable to a wide range of Bayesian games. Here, we adapt that model to our particular inferential context.[12]

Channeling Loewenstein et al.'s (2003) model of intrapersonal projection bias, Gagnon-Bartsch et al. (2021a) assume that each agent i perceives the private value of another agent j as a convex combination between j's true value and i's own value; that is, i believes j's private value is $\hat{t}_j(t_i) \equiv \alpha t_i + (1-\alpha)t_j$, with $\alpha \in [0, 1)$. The parameter α captures the "degree of projection": $\alpha = 0$ is the rational benchmark, while $\alpha \to 1$ represents the extreme case where an agent believes that others share his exact taste. For tractability, we assume the degree of projection is identical across agents.

Perceptions of the Taste Distribution. The convex-combination specification above implies that agent i's perception of others' private values is described by the random variable

$$\widehat{T}(t_i) \equiv \alpha t_i + (1-\alpha)T, \tag{1}$$

where $T \sim F$ is the true random variable describing private values. Hence, each agent i perceives a distribution of tastes that, relative to reality, is overly concentrated around his own taste, t_i.[13]

This formulation of projection pins down the perceived distributions held by projecting agents, $\{\widehat{F}(\cdot|t)\}_{t\in\mathcal{T}}$, in terms of the true distribution, F, and the projection parameter, α. Each agent perceives a distribution with the same shape as F, but with the probability mass compressed around his own value. The support of this distribution is also compressed when $\mathcal{T}$ is bounded: Equation (1) implies that an agent with type t has a perceived support of $\widehat{\mathcal{T}}(t) \equiv [\underline{t}(t), \bar{t}(t)] \subset \mathcal{T}$, where $\underline{t}(t) \equiv \alpha t + (1-\alpha)\underline{t}$ and $\bar{t}(t) \equiv \alpha t + (1-\alpha)\bar{t}$.[14] Moreover, this type's perceived CDF is

[11] As emphasized by Gagnon-Bartsch et al. (2021b), subjectively rational inattention (with respect to a misspecified model) may lead an agent to attend to data he deems sufficient for updating his beliefs (e.g., current demand at a given price), while forgoing careful attention to additional data (e.g., pricing strategy). Indeed, when the seller's cost is subject to noise that is unobserved by buyers, our results are "attentionally stable" in the sense of Gagnon-Bartsch et al. (2021b); that is, a consumer need not confront data that reveals his model's misspecification.

[12] For a review of the motivating evidence, see Gagnon-Bartsch et al. (2021a) and the references therein.

[13] Gagnon-Bartsch et al. (2021a) discusses how this approach naturally extends to cases where values are correlated or agents are asymmetric. Since we focus on settings with i.i.d. types, we forgo these elaborations.

[14] Our qualitative results do not hinge on misperceptions of the support per se. They would continue to hold if each type's perceived distribution were approximately the same as Equation (1) but modified to assign a small yet positive probability to types in $\mathcal{T} \setminus \widehat{\mathcal{T}}(t_i)$.

$$\widehat{F}(x|t) = \Pr\left(\widehat{T}(t) \leq x\right) = \begin{cases} 0 & \text{if } x < \underline{t}(t) \\ F\left(\frac{x-\alpha t}{1-\alpha}\right) & \text{if } x \in [\underline{t}(t), \bar{t}(t)] \\ 1 & \text{if } x > \bar{t}(t). \end{cases} \tag{2}$$

These perceived distributions inherit our assumptions on F: each $\widehat{F}(\cdot|t)$ admits a smooth, positive log-concave density $\hat{f}(x|t) = \left(\frac{1}{1-\alpha}\right) f\left(\frac{x-\alpha t}{1-\alpha}\right)$ for $x \in \widehat{\mathcal{T}}(t)$. Intuitively, under these biased perceptions, a projecting agent thinks the average taste is closer to his own than it really is, and underestimates the variance in tastes.

Higher-Order Beliefs. We assume each projector exhibits *naivete* about his bias by neglecting that he and others misperceive the distribution of tastes; therefore, he fails to appreciate that others form discrepant perceptions of this distribution. An agent with private value t thus thinks that (i) all others believe that private values are distributed according to $\widehat{F}(\cdot|t)$, and (ii) this mutual perception is common knowledge. This assumption is motivated by the idea that people who are inattentive to their own projection bias are likely inattentive to others' projection bias as well; this further differentiates our model from Madarász et al. (2023), where agents project onto others and simultaneously anticipate others' projection onto them.[15] Naivete also differentiates our model from rational models in which an agent's own taste shapes his beliefs about others' tastes; e.g., correlated private values or uncertainty about F.[16]

Solution Concept. We apply Gagnon-Bartsch et al.'s (2021a) "Naive Bayesian Equilibrum" concept to our setting. Aside from misperceptions about F (and about others' misperceptions of F), we assume projecting agents are otherwise rational and believe others are rational. Each player maximizes his expected payoff according to his distorted beliefs and the presumption that others share his misspecified model. Therefore, each player i plays his strategy in the Bayesian Nash Equilibrium (BNE) of the "perceived game" in which $\widehat{F}(\cdot|t_i)$ is indeed the commonly-known taste distribution. The resulting profile of strategies is a *Naive Bayesian Equilibrium (NBE)*.

To formalize this concept within our setting, suppose the true symmetric game under consideration is Γ with an action space $\mathcal{A} \subseteq \mathbb{R}$. Let $\Gamma(\widehat{F})$ denote that same game when the type distribution is $\widehat{F}$ instead of F; all other elements of $\Gamma(\widehat{F})$ are identical to Γ. A player with type t thinks the game is $\Gamma(\widehat{F}(\cdot|t))$ and presumes that players will follow a BNE of $\Gamma(\widehat{F}(\cdot|t))$. Let $\tilde{\sigma}(\cdot|t)$ denote a

[15] Although studies on the false-consensus effect rarely elicit second-order beliefs, the few that do, e.g. Egan et al. (2014), find that people overestimate how many share their second-order beliefs, which is consistent with naivete. Of course, our assumption of complete naivete is likely an oversimplification; in the domain of information projection, Madarász et al. (2023) find that experimental subjects are partially aware of others' biases.

[16] Dawes (1989, 1990) argues that the type-dependent beliefs found in experimental studies may reflect rational uncertainty about others' tastes rather than projection bias. However, studies responding to this critique find that subjects' perceptions systematically overweight their own preference relative to information about others' preferences when making predictions about others (e.g., Krueger and Clement, 1994). Engelmann and Strobel (2012) and Ambuehl et al. (2021) similarly find that a false-consensus bias remains despite access to information about others' choices.

symmetric pure strategy profile within the perceived game $\Gamma(\widehat{F}(\cdot|t))$.

Definition 1. *A symmetric strategy profile $\hat{\sigma} : \mathcal{T} \to \mathcal{A}$ is a symmetric Naive Bayesian Equilibrium (NBE) of Γ if, for all $t \in \mathcal{T}$, there exists a symmetric strategy profile $\tilde{\sigma}(\cdot|t) : \widehat{\mathcal{T}}(t) \to \mathcal{A}$ that is a BNE of $\Gamma\big(\widehat{F}(\cdot|t)\big)$ and $\hat{\sigma}(t) = \tilde{\sigma}(t|t)$.*

To provide some intuition, each player with taste t introspects about others' behavior within his perceived game, and this process leads him to a conjectured BNE strategy profile, $\tilde{\sigma}(\cdot|t)$, of that game.[17] He then follows the strategy prescribed by this conjectured equilibrium; i.e., he takes action $\tilde{\sigma}(t|t)$. A NBE is the strategy profile that emerges when each player engages in this reasoning.

Note that a BNE strategy in our setting is just a map from a buyer's type, t, and expectation of quality, $\hat{\omega}$, to a binary purchase decision. A projecting buyer correctly understands another buyer's strategy conditional on their type and expectation. However, the aggregate behavior that the projecting buyer observes will depend on the distribution of t and $\hat{\omega}$ in the market. He thus misinterprets aggregate behavior due to two mistakes about these distributions: (i) he misperceives the distribution of types, t, acting in the market; and (ii) he mispredicts others' quality expectations, $\hat{\omega}$, since he neglects that those with different types employ inferential strategies different from his.

3 Static Model

We begin by showing how taste projection distorts beliefs in a static model. As we discuss below, this model can be interpreted as the steady-state equilibrium of our dynamic model (in Section 4) when the price is held constant across periods. The analysis here allows us to establish key implications of mislearning due to taste projection in a relatively simple way; it also demonstrates that the comparative statics that will arise in the dynamic model below robustly emerge in the steady-state as well. Namely, an agent's perceived quality is (i) decreasing in his private taste, and (ii) increasing in the price. Furthermore, the perceived total valuations of agents in equilibrium are excessively similar to each other, leading to a volatile market demand that overreacts to price changes.

Building on the setup from Section 2.1, a continuum of potential buyers with unit mass face a fixed price p. A fraction $\lambda \in (0, 1)$ of the agents privately observe the realization of $S \sim G(\cdot|\omega)$ and the remaining fraction $1 - \lambda$ do not. The "uninformed agents"—those who do not observe the signal—attempt to extract this information from the equilibrium level of demand.

The steady-state equilibrium follows a logic similar to a rational-expectations equilibrium (e.g., Grossman, 1976; Grossman and Stiglitz, 1980), except agents wrongly use their misspecified models to extract signals. Suppose the fraction of agents who buy is $d \in [0, 1]$. Each uninformed agent

[17] Because $\widehat{F}(\cdot|t)$ inherits our assumptions on F, existence of such a BNE in the perceived game $\Gamma(\widehat{F}(\cdot|t))$ follows from the existence of a BNE in the original game Γ.

follows an inference rule that maps d into an expectation over ω, and then buys the good if their expected valuation exceeds p. In equilibrium, agents' inferences about ω must be consistent with the observed quantity demanded, and this quantity must in turn be consistent with agents' inferences.

We now derive the equilibrium. Informed agents base their buying decisions entirely on s, as they know there is nothing more to learn. Thus, an informed agent with taste t buys if $\bar{\omega}(s)+t \geq p$, and the informed demand is $D^I(p;\bar{\omega}(s)) \equiv \Pr\left[\bar{\omega}(s)+T \geq p\right] = 1-F\left(p-\bar{\omega}(s)\right)$. We say that the pair (p,s) *admits interior demand* when $D^I(p;\bar{\omega}(s)) \in (0,1)$.

Uninformed agents infer $\bar{\omega}(s)$ from the aggregate quantity demanded, d. To build intuition, we first describe agents' inferences in the rational benchmark. Let $\hat{\omega}(d)$ denote the inferred value of $\bar{\omega}(s)$ upon observing d. Demand among the uninformed is thus $\Pr[\hat{\omega}(d)+T \geq p] = 1-F(p-\hat{\omega}(d))$, and the total demand is

$$d = \lambda \cdot \underbrace{\big(1-F\big(p-\bar{\omega}(s)\big)\big)}_{\text{Demand among the informed}} + (1-\lambda)\cdot \underbrace{\big(1-F\big(p-\hat{\omega}(d)\big)\big)}_{\text{Demand among the uninformed}}. \tag{3}$$

We require that $\hat{\omega}(d)$ is Bayes-rational given an agent's model. Hence, in the rational benchmark—where players share common knowledge of F—the unique symmetric inference rule is $\hat{\omega}(d) = p-F^{-1}(1-d)$. When following this rule, the observed quantity demanded d is such that uninformed agents infer $\hat{\omega}(d)=\bar{\omega}(s)$ and hence behave as informed agents. This follows from the fact that, in equilibrium, d reveals the marginal buyer's type.[18]

This strategy of identifying others' information off of the inferred marginal type leads projectors astray since they misinfer the type of the marginal buyer. More specifically, a projecting agent thinks the market is in the rational equilibrium described above, and draws inferences following that logic. They do so, however, using their misspecificed model. A buyer with taste t_i thinks the demand function among informed agents is $\widehat{D}^I(p;\bar{\omega}(s)|t_i) \equiv 1-\widehat{F}\big(p-\bar{\omega}(s)|t_i\big)$. Furthermore, due to naivete, he thinks others (i) share his perception of F, and hence of the demand function, and (ii) will draw the same inference as him. Thus, an agent with taste t_i thinks the rational symmetric inference rule is $\hat{\omega}(d|t_i)$ and that, in equilibrium, $\hat{\omega}(d|t_i)$ must satisfy

$$d = \lambda \cdot \underbrace{\Big(1-\widehat{F}\big(p-\bar{\omega}(s)|t_i\big)\Big)}_{\text{Perceived demand among the informed}} + (1-\lambda)\cdot \underbrace{\Big(1-\widehat{F}\big(p-\hat{\omega}(d|t_i)|t_i\big)\Big)}_{\text{Perceived demand among the uninformed}}. \tag{4}$$

Consequently, an agent with taste t_i comes to believe that the value of $\bar{\omega}(s)$ is

$$\hat{\omega}(d|t_i) = p-\widehat{F}^{-1}(1-d|t_i). \tag{5}$$

[18] For instance, if 30% of the market buys at p, then the marginal buyer has a private value at the 70$^{\text{th}}$ percentile of F. Thus, rational uninformed agents simply choose to buy if their taste is above the 70$^{\text{th}}$ percentile and decline otherwise.

This inferential strategy would correctly extract others' information if agent i's misspecified model were correct (i.e., if $T \sim \widehat{F}(\cdot|t_i)$ and agents shared this belief).[19]

The misinference described above involves two distinct errors. One stems from an error in first-order beliefs: agent i's conjectured equilibrium condition wrongly posits that tastes are distributed according to $\widehat{F}(\cdot|t_i)$ instead of F. Additionally, due to naivete, agent i's erroneous second-order beliefs cause him to think others draw the same inference as him, $\hat{\omega}(d|t_i)$, since he neglects that others employ discrepant models.

In truth, the demand among uninformed agents arises from each type of agent acting on their distinct inference. The equilibrium quantity demanded is then the value of d solving

$$d = \lambda \cdot D^I\left(p; \bar{\omega}(s)\right) + (1-\lambda) \cdot \underbrace{\Pr[\hat{\omega}(d|T) + T \geq p]}_{\text{Demand from Uninformed Agents}}, \tag{6}$$

where $\hat{\omega}(d|t)$ is given by (5) for each $t \in \mathcal{T}$. This equilibrium quantity, call it d^*, pins down the profile of agents' perceptions of $\bar{\omega}(s)$. We denote this profile by $\hat{\omega}(t)$; that is, $\hat{\omega}(t) = \hat{\omega}(d^*|t)$. The following proposition establishes two central properties of misinference under taste projection.

Proposition 1. *Suppose (p, s) admits interior demand. For any $\alpha > 0$, there exists a unique equilibrium profile of beliefs, and it has the following properties:*

1. *$\hat{\omega}(t)$ is strictly decreasing in t. Moreover, there exists an interior type $\tilde{t}$ such that agents with $t > \tilde{t}$ underestimate ω while those with $t < \tilde{t}$ overestimate ω.*

2. *For each type $t \in \mathcal{T}$, $\hat{\omega}(t)$ is strictly increasing in p.*

Part 1 of Proposition 1 establishes that quality perceptions are inversely related to tastes. If agent i has a high private taste, he expects that others do too, exaggerating the fraction of people who buy conditional on price p and belief $\bar{\omega}(s)$. Accordingly, the actual demand at price p looks rather weak and, to rationalize it, he must infer a relatively low common value. Conversely, if agent i has a low private taste, he will infer a relatively high common value. In other words, the interpretation of a good's popularity is in the eye of the beholder.[20]

Gagnon-Bartsch and Bushong (2023) find evidence of this prediction in a social-learning experiment explicitly designed to study how inaccurate perceptions of others' tastes may distort inference. Subjects stated their updated beliefs about the amount of money on gift cards for various businesses after observing the choices of others who received private signals about the cards' nominal values. The results show that observers formed lower estimates of a card's nominal value when the card was

[19]Note that $\widehat{F}^{-1}(1-d|t_i) \to F^{-1}(1-d)$ for all t_i as $\alpha \to 0$. Hence, each agent's inference collapses to the common rational inference as projection vanishes.

[20]It is worth noting that, as shown in the proof of Proposition 1, perceived *total* valuations, $\hat{\omega}(t)+t$, are still increasing in t even though $\hat{\omega}(t)$ is decreasing in t.

for a business that they personally found more desirable. That is, subjects interpreted others' actions as if others shared their idiosyncratic tastes over the businesses. Gagnon-Bartsch and Bushong (2023) also directly elicited subjects' beliefs about others' tastes and find that subjects indeed overestimated how similar others' preferences were to their own. This suggests that taste projection may be the mechanism driving discrepant social learning in their study.

Where is the divide between types who overestimate quality and those who underestimate it? As noted above, inference in this setting stems from identifying the valuation of the marginal consumer. The nature of projectors' misinference can thus be understood from how they misidentify the marginal type. Suppose that in equilibrium a fraction z of consumers turn down the good. The marginal type thus has a private value t^* at the z^{th} percentile of the taste distribution. An uninformed consumer tries to deduce t^* since this would reveal $\bar{\omega}(s)$ via the indifference condition $t^* = p - \bar{\omega}(s)$. However, a projector misperceives the private value at each percentile other than his own. To see this, let $\hat{t}(z|t_i)$ be the perceived type at the z^{th} percentile according to an agent with taste t_i, and let $t^*(z)$ denote the true type. From (2), $\hat{t}(z|t_i)$ solves

$$z = \widehat{F}(\hat{t}(z|t_i)|t_i) = F\left(\frac{\hat{t}(z|t_i) - \alpha t_i}{1-\alpha}\right) \;\;\Rightarrow\;\; \hat{t}(z|t_i) = \alpha t_i + (1-\alpha)t^*(z). \tag{7}$$

Reflecting the idea that projectors think others' values are compressed around their own, type t_i's perception of the type at the z^{th} percentile is shifted toward his own. This recasts the intuition from above: those with high private values overestimate the marginal type, and thus underestimate the good's quality; those with low private values do the opposite. Furthermore, this means that a projector who is at the z^{th} percentile himself—who has a taste matching that of the truly marginal type—is the unique type who infers $\bar{\omega}(s)$ correctly. To summarize: (i) $\hat{\omega}(t^*) = \bar{\omega}(s)$ where $t^* = p - \bar{\omega}(s)$ is the truly marginal type; (ii) $\hat{\omega}(t) < \bar{\omega}(s)$ for all agents with $t > t^*$; and (iii) $\hat{\omega}(t) > \bar{\omega}(s)$ for all agents with $t < t^*$.

Part 2 of Proposition 1 shows that, irrespective of their private taste, projecting agents form higher perceptions of the common value when p is higher. This stems from the fact that projectors underestimate the heterogeneity in others' private values and, therefore, underestimate the fraction of types who would remain in the market at a higher price. If the price were to increase, a projector would see more remain than he expected; to rationalize this discrepancy, he must then infer a higher quality. Figure 1 depicts this intuition. First, note that a projector's inferred quality $\hat{\omega}$ is such that his perceived demand function given $\hat{\omega}$, $\widehat{D}(\cdot;\hat{\omega}|t)$, passes through the market outcome, (p, d). If the price increased from p' to p'', the new quantity demanded would be determined by the true demand curve, $D(\cdot;\bar{\omega}(s))$. This new quantity, however, would be inconsistent with the projectors' demand curve that rationalized the outcome at p': since projectors underestimate heterogeneity, their perceived demand curve is a counter-clockwise rotation of $D(\cdot;\bar{\omega}(s))$ (see Johnson and Myatt, 2006) and is

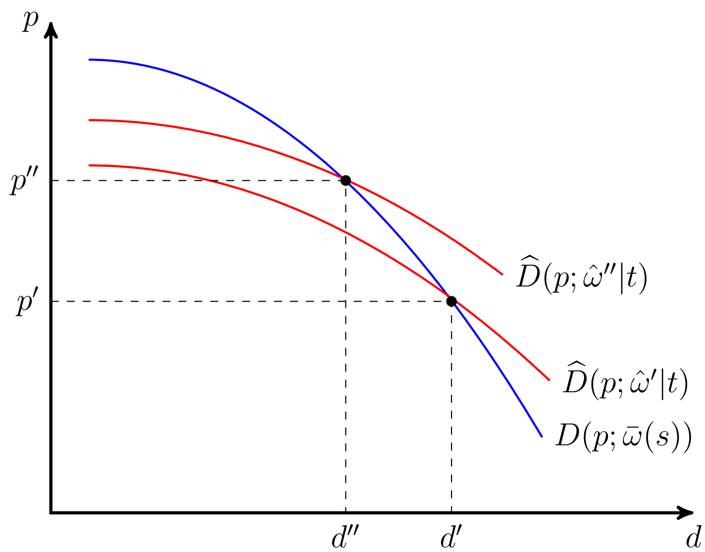


Figure 1: *True and perceived equilibrium demand functions.*

thus more price elastic. Hence, to rationalize the quantity demanded at price p'', a projector must form a higher expectation of ω, consistent with an outward shift of his perceived demand curve.

While the results of Proposition 1 hold more generally, they are particularly transparent given our assumption that $u(\omega, t) = \omega + t$.[21] In this case,

$$\hat{\omega}(t) = (1 - \alpha)\bar{\omega}(s) + \alpha(p - t). \tag{8}$$

The degree of projection, α, drives both the positive distortionary effect of p and the negative distortionary effect of an individual's taste. Furthermore, an uninformed agent's perceived *total* value of the good is $\hat{\omega}(t) + t = (1 - \alpha)(\bar{\omega}(s) + t) + \alpha p$. Thus, as α increases, a projector's idiosyncratic taste t has less influence on their perceived valuation. Importantly, this implies that the perceived valuations among uninformed agents exhibit less variation than they would under rational inference.

Proposition 2. *Suppose (p, s) admits interior demand. For any $\alpha > 0$, the (mis)perceived valuations of agents in the steady-state have diminished variance relative to the rational benchmark.*

Proposition 2 reveals a sense in which taste projection is self-fulfilling: when agents initially believe that idiosyncratic tastes are more similar than they really are, their distorted inferences lead to perceived valuations that are, in fact, more similar than they ought to be. In other words, the agents' initial misperception of the environment generates data that confirms their misperception.

Given the distortions in beliefs described above, what price would maximize a seller's profits? In this particular setting, the optimal static price under projection is the same as under rational learning. This happens because, as noted above, the buyer with a type matching that of the marginal buyer

[21] Our proofs of Propositions 1 and 2 in Appendix A establish these results for more general utility functions.

in the rational benchmark is still marginal under projection. However, the reason that projection affects beliefs but not market outcomes here is due to the particular setting—namely, there is no opportunity to vary prices over time and consumers have unit demand. Although this setting is ideal for developing intuitions on why and how projection distorts beliefs, the following sections show that relaxing these features will cause biased beliefs to distort market outcomes. In particular, the next section examines how the seller could benefit from dynamic pricing. Indeed, Proposition 2 hints at the fact that in a dynamic setting the demand among misinformed consumers will overreact to price changes; intertemporal price dispersion allows a profit-maximizing seller to exploit this overly volatile demand.

4 Dynamic Model

We now turn to a dynamic version of our model. In each period $n = 1, 2, \ldots, N$, a unit mass of new consumers with tastes independently drawn from F enters the market. Each consumer in Generation n simultaneously chooses once-and-for-all whether to buy the good at price p_n and then exits; d_n denotes the fraction of these consumers who buy. In each generation $n \geq 2$, (i) all individuals observe the price and aggregate demand from the previous generation, (p_{n-1}, d_{n-1}), and (ii) a fraction $\lambda \in (0, 1)$ privately observe s. Thus, $1 - \lambda$ uninformed consumers in each generation $n \geq 2$ engage in social learning while the informed consumers simply follow the signal.

In period 1, consumers must make decisions based solely on their private information. To simplify matters, we assume all consumers in period 1 observe s. In this way, our analysis examines the extent to which the private information held by initial consumers, s, is successfully transmitted to later consumers. There are at least two interpretations of this assumption: (i) early consumers have greater access to information than later ones (e.g., initial advertising or "hands-on" promotions spread information more widely early on); (ii) the market begins in the steady-state equilibrium derived in Section 3. Under the second interpretation, our results here describe the short-run dynamics of beliefs and market demand when price changes move the market out of the steady state. This assumption also simplifies the analysis by ensuring that the seller does not have an informational advantage over buyers, thereby neutralizing any incentive for the seller to use prices to signal quality (see the discussion at the end of Section 2.1).[22]

Rational learning is straightforward. Since a continuum of agents act in each period, the aggregate demand from the previous period fully reveals the signal when there is common knowledge of F (and of rationality). While agents learn immediately in this rational benchmark, projectors do not

[22] While the assumption that all consumers in Generation 1 are privately informed simplifies the analysis in various ways, it does not significantly influence the results. For instance, if a fraction $\lambda < 1$ of consumers observe s in each period $n = 1, 2, \ldots$ and face a fixed exogenous price, then the environment corresponds to the dynamic analog of the static model in Section 3: beliefs and behavior converge to the steady-state values described in Section 3.

since they wrongly extract the signal as if it were common knowledge that $T \sim \widehat{F}(\cdot|t_i)$.[23]

Below, we primarily focus on the two-period case where the intuitions are most transparent. Within the two-period model, Section 4.1 first characterizes how beliefs and market demand evolve under exogenously given prices; Section 4.2 then analyzes dynamic monopoly pricing. Section 4.3 describes how the key effects of projection extend beyond the two-period case.

4.1 Beliefs and Demand with Two Periods

We begin by analyzing the inferences of uninformed consumers in period 2 upon observing (p_1, d_1). Here, we take p_1 to be any price such that (p_1, s) admits interior demand.

In period 1, aggregate demand matches the rational benchmark: $d_1 = D^I(p_1; \bar{\omega}(s)) = 1 - F(p_1 - \bar{\omega}(s))$.[24] In period 2, an individual with taste t thinks that when buyers in period 1 have expectations equal to $\hat{\omega}$, their demand is

$$\widehat{D}^I(p_1; \hat{\omega}|t) = 1 - \widehat{F}(p_1 - \hat{\omega}|t) = 1 - F\left(\frac{p_1 - \hat{\omega} - \alpha t}{1 - \alpha}\right). \tag{9}$$

This individual will then infer a value of $\hat{\omega}$ that solves $\widehat{D}^I(p_1; \hat{\omega}|t) = d_1$. Denoting this value by $\hat{\omega}_2(t)$, the previous condition yields

$$\hat{\omega}_2(t) = (1 - \alpha)\bar{\omega}(s) + \alpha(p_1 - t). \tag{10}$$

Notice that the misinferences among observers in this dynamic context exhibit the same properties described in Propositions 1 and 2 from the static model. Indeed, (10) exactly matches the steady-state perceptions derived in Equation (8). These perceptions are decreasing in an observer's taste, increasing in the price, and give rise to perceived total valuations that exhibit too little heterogeneity.

Given these distortions in beliefs, we can further characterize how the market demand in period 2 under biased learning differs from that under and rational learning. In the rational case, this demand would match that of informed consumers; i.e., $D^I(p_2; \bar{\omega}(s)) = 1 - F(p_2 - \bar{\omega}(s))$. To derive demand in the biased case, first notice that if we let $\bar{\omega}_2 \equiv (1 - \alpha)\bar{\omega}(s) + \alpha p_1$ denote the "taste-independent" (mis)perception of $\bar{\omega}(s)$ among uninformed consumers in period 2, then (10) implies that the perceived total valuation of such a consumer with taste t_i is $u\left(\hat{\omega}_2(t_i), t_i\right) = \bar{\omega}_2 + (1 - \alpha)t_i$.

[23] Although we assume agents observe the market outcome from only the previous period, this data is sufficient for rational agents to learn the signal. If biased agents were to observe longer histories of outcomes, they may see data inconsistent with their misspecified models. This is because we assume there is a single signal in the market and no other source of noise. We could alternatively avoid such inconsistencies by adding further sources of noise (e.g., prices or demand subject to shocks), but this would significantly complicate the analysis while adding limited additional insight.

[24] This follows from our assumption that all consumers in period 1 are informed (i.e., they observe s).

The demand among uninformed consumers in period 2 is thus

$$D^U(p_2;\bar{\omega}_2) \equiv \Pr\left[u\left(\hat{\omega}_2(T),T\right) \geq p_2\right] = 1 - F\left(\frac{p_2 - \bar{\omega}_2}{1-\alpha}\right), \tag{11}$$

and total demand is

$$D(p_2;\bar{\omega}_2,\bar{\omega}(s)) \equiv \lambda D^I(p_2;\bar{\omega}(s)) + (1-\lambda)D^U(p_2;\bar{\omega}_2). \tag{12}$$

We can now show that the demand function in period 2 is locally more price-elastic than the rational one (Johnson and Myatt, 2006). More specifically, it is a counter-clockwise rotation of the rational one, and the rotation point is the market outcome from the previous period, (p_1, d_1).

Proposition 3. *Suppose (p_1, s) admits interior demand and consider any $\alpha > 0$. The demand curve in the second period under biased learning, $D(p_2;\bar{\omega}_2,\bar{\omega}(s))$, is a counter-clockwise rotation of the demand curve under rational learning, $D^I(p_2;\omega(s))$, about the point (p_1, d_1).*

It is clear that $\alpha > 0$ implies that $D^U(p_2;\bar{\omega}_2)$—and thus $D(p_2;\bar{\omega}_2,\bar{\omega}(s))$—is more sensitive to p_2 than the demand among rational observers with the same beliefs (see Figure 2). The rationale builds from intuitions developed in the static case: in period 2, perceptions of $\bar{\omega}(s)$ are declining in consumers' private values, and the buyer with a private value equal to that of the marginal type from period 1, denoted t_1^*, is the unique uninformed type who infers $\bar{\omega}(s)$ correctly. Those with $t > t_1^*$ see a weaker demand in period 1 than anticipated given $\bar{\omega}(s)$ and consequently underestimate ω. In contrast, those with $t < t_1^*$ see a stronger demand than anticipated given $\bar{\omega}(s)$ and overestimate ω.

An implication of Proposition 3 is that, relative to the rational benchmark, the demand in period 2 overreacts to price changes. If $p_2 > p_1$, then only those types with overly pessimistic beliefs will be served in period 2, and the quantity demanded will thus fall below the rational benchmark at p_2. If $p_2 < p_1$, then those with overly optimistic beliefs will be served—the marginal type will be among this contingent—and hence the quantity demanded will exceed the rational benchmark. Social learning under taste projection therefore offers a novel explanation for overreaction to price changes, thereby complementing other existing—yet conceptually distinct—explanations. For instance, a change in the price could momentarily increase attention or salience to the price (Bordalo et al., 2013, 2020). Or consumers with a "taste for bargains" may experience additional elation when buying at a price below some reference level (e.g., the previous price), thereby leading more to buy when the new price feels like a "deal" (Jahedi, 2011; Armstrong and Chen, 2020).

4.2 Optimal Monopoly Pricing with Two Periods

We now analyze how a sophisticated seller optimally sets prices over time when facing taste-projecting consumers. The seller chooses a price p_n, $n \in \{1, 2\}$, at the start of each period to

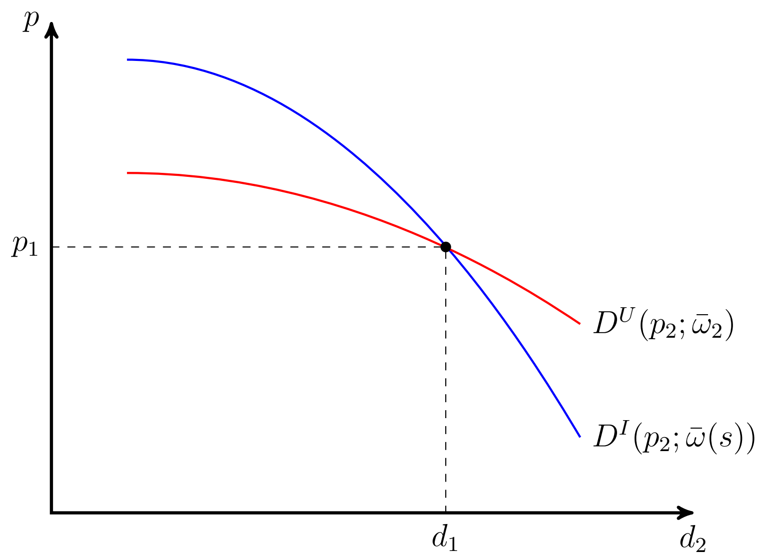


Figure 2: *Demand functions of the informed and uninformed in period 2.*

maximize profits

$$\Pi \equiv p_1 D^I(p_1; \bar{\omega}(s)) + p_2 D(p_2; \bar{\omega}_2, \bar{\omega}(s)), \tag{13}$$

subject to biased inferences given by the value $\bar{\omega}_2$ characterized above.[25] In order for the following generation to draw a well-defined inference from the preceding one, we require the seller to serve a positive fraction of consumers in each period. We operationalize this by imposing a price ceiling that is arbitrarily close to the valuation of the highest informed type: $\bar{p} \equiv \bar{\omega}(s) + \bar{t} - \kappa$ for some small $\kappa > 0$.[26]

Let p_n^* denote the seller's profit-maximizing price in period n. Under rational learning, all consumers will correctly infer s, and the seller essentially faces an identical market of informed consumers in each period. Let p^M denote the static optimal monopoly price when facing informed consumers. The optimal price path in the rational benchmark (i.e., $\alpha = 0$) is to simply charge $p_n^* = p^M$ for all n. As we emphasize below, this is not so with projecting consumers (i.e., $\alpha > 0$).

Taste projection among consumers introduces dynamic pricing incentives for the seller. Indeed, since the current price inflates the beliefs of consumers in later periods, the seller may benefit from increasing today's price—at the cost of losing immediate sales—in order to increase perceptions and demand among future consumers. The benefit from such manipulation is clearly suggested by the distorted beliefs formed in Generation 2, as described in (10). The private value of the marginal

[25] For simplicity, we abstract from the seller discounting future profits. Our results would continue to hold if the seller exponentially discounted future profits with a discount factor $\delta \in (0, 1)$.

[26] This price ceiling will have little effect on projectors' beliefs and behavior since projectors can never be induced to have a willingness to pay above the highest informed type. The price ceiling is also not consequential for our qualitative results: the optimal price path still involves an inflated price in period 1 and a subsequent price reduction regardless of whether p_1 is at the ceiling or not. Furthermore, for every value of α, there exists a value $\bar{\lambda}$ such that $\lambda > \bar{\lambda}$ guarantees an interior solution to the seller's problem, rendering the ceiling irrelevant.

type in Generation 1 determines the threshold in the taste distribution where $\hat{\omega}_2(t)$ switches from overestimating quality to underestimating it. As this threshold is increasing in p_1, a higher p_1 will result in a larger share of individuals in Generation 2 who overestimate quality.

But is it worthwhile for the seller to forego sales today in order to boost demand in the future? The answer is unambiguously yes. To provide intuition, consider two pricing strategies: (i) constant pricing, where $p_1 = p_2 = p^M$, and (ii) declining prices such that $p_1 = p^M + \epsilon$ and $p_2 = p^M - \epsilon$ for some $\epsilon > 0$. The first strategy generates profits identical to the rational benchmark. While the second strategy generates diminished sales in period 1 relative to the rational benchmark, it generates a disproportionate expansion in period 2. This happens because the demand curve in Generation 2 is a counter-clockwise rotation around p_1 of the demand curve from the previous generation. Locally, a small reduction of p_2 below p^M leads to a greater expansion in period-2 sales compared to the contraction of period-1 sales induced by a commensurate increase of p_1 above p^M. This is because those types who would have been submarginal in period 1 hold inflated perceptions; hence, a price cut attracts an exaggerated share of consumers (as in Proposition 3). As a result, the profits gained in period 2 more than offset those lost in period 1.[27] This intuition holds more generally.

Proposition 4. *Suppose that (p^M, s) admits interior demand.*

1. *For any $\alpha > 0$, we have $p_1^* > p^M$ and $p_1^* > p_2^*$.*
2. *The seller's profit under the optimal price path is increasing in α and decreasing in λ.*

Intuitively, as α increases, there is greater scope to manipulate beliefs, thereby increasing the seller's profit above the rational benchmark. The seller's profit is instead decreasing in λ: with fewer uninformed agents in the market, it becomes more costly to deviate from the rational-benchmark price. Additionally, although p_1^* always exceeds p^M, the relationship between p_2^* and p^M depends on the degree of projection. When α is low and projectors' beliefs are only mildly distorted by p_1, the seller optimally chooses $p_2 < p^M$ to induce a large share of overoptimistic types to buy. When α is high and beliefs are strongly distorted by p_1, then even a $p_2 > p^M$ can induce these types to buy.

Pricing under projection clearly harms consumers in period 1 since $p_1^* > p^M$. But it also harms some consumers in period 2: beliefs are manipulated in a way that leads some low-valuation consumers to buy at a price they would refuse under rational learning. While some of this harm simply represents a transfer to the seller, sufficiently strong projection can induce consumers who value the good less than its cost (i.e., zero in this case) to buy it. This is clearly inefficient.

Proposition 5. *Suppose that (p^M, s) admits interior demand and consider the behavior of uninformed consumers in period 2.*

[27] By a similar logic, choosing $p_2 > p_1$ is particularly costly for the seller, as this would exclude optimistic consumers while targeting just the pessimistic ones.

1. *For any $\alpha > 0$, under the profit-maximizing price path, there exists a positive measure of types who buy and overpay; for these types, $\bar{\omega}(s) + t < p_2$.*

2. *If there exist types with truly negative valuations, i.e., $\bar{\omega}(s) + \underline{t} < 0$, then there exists a threshold $\tilde{\alpha}$ such that for $\alpha > \tilde{\alpha}$ the profit-maximizing price path induces inefficient adoption: there exists an interval of types t who buy despite $\bar{\omega}(s) + t < 0$.*

An implication of this result is that the seller's optimal pricing scheme always induces excessive take-up among uninformed buyers, consistent with familiar notions of herding or bandwagon effects in markets.[28] Such excessive take-up indeed leads some uninformed consumers to overpay. Figure 3 shows the demand curves among informed (blue) and uninformed consumers (red) in period 2. The demand curve among informed consumers, $D^I(p;\bar{\omega}(s))$, reflects the rational valuation of the marginal buyer for any level of market coverage d. The demand curve among uninformed consumers, $D^U(p;\bar{\omega}_2)$, instead reflects the willingness to pay of the marginal consumer given d. Thus, for any d, the vertical gap between the red and blue curves reflects the wedge between the marginal uninformed consumer's willingness to pay and his true valuation. Manipulative pricing under projection causes a range of uninformed types to buy the good when they should, in fact, abstain given p_2^*: the rational level of demand at p_2^* is d_2^I, yet a market of projectors would demand a quantity $d_2 > d_2^I$. Projectors' consumer surplus is no longer simply the area below their demand curve and above the price, since all consumption beyond d_2^I involves overpaying. Instead, projectors' surplus is the area above p_2^* yet below their valuation curve (area in blue) *minus* the area below p_2^* yet above their valuation curve (area in red). Moreover, the dark red triangle displays a case where such over-adoption is inefficient since consumers with negative valuations are lured into buying.

To further elucidate the pricing and welfare effects of projection, suppose T is uniform on $[\underline{t}, \bar{t}]$. In this case, the demands of informed and uninformed agents are

$$D^I\big(p;\bar{\omega}(s)\big) = \frac{\bar{\omega}(s) + \bar{t} - p}{\bar{t} - \underline{t}} \quad \text{and} \quad D^U(p;\bar{\omega}_2) = \frac{\bar{\omega}_2 + (1-\alpha)\bar{t} - p}{(1-\alpha)(\bar{t} - \underline{t})}, \tag{14}$$

respectively, where $\bar{\omega}_2 = (1-\alpha)\bar{\omega}(s) + \alpha p_1$. It is straightforward to show that the interior solution is such that $p_1^* > p^M > p_2^*$. Panel (a) of Figure 4 shows how each p_n^* changes with α.[29] Intuitively, $p_n^* \to p^M$ for both $n = 1, 2$ as $\alpha \to 0$. As α increases, p_1 has a stronger positive effect on the beliefs of Generation 2, and hence p_1^* increases in α. By contrast, p_2^* is not monotone in α. Since the consumers who would be submarginal at p_1^* are those with inflated beliefs, p_2^* will necessarily fall below p_1^*. Moreover, when α is small, the perceived valuations of consumers in Generation 2

[28] It is straightforward to show that the marginal uninformed type in period 2, $\hat{t}_2$, is strictly below the marginal informed type, t_2^*, and the interval of uninformed types who wrongly buy the good has measure $t_2^* - \hat{t}_2 = \frac{\alpha(p_1^* - p_2^*)}{1-\alpha} > 0$.

[29] In this example, $\bar{t} = 10$, $\underline{t} = -10$, $\bar{\omega}(s) = 0$. We plot outcomes for $\alpha \leq 2/3$ since this is the region that admits an interior solution (shown in the figure). For $\alpha > 2/3$, we necessarily have a corner solution at which the seller sets p_1 at the price ceiling (see footnote 26).

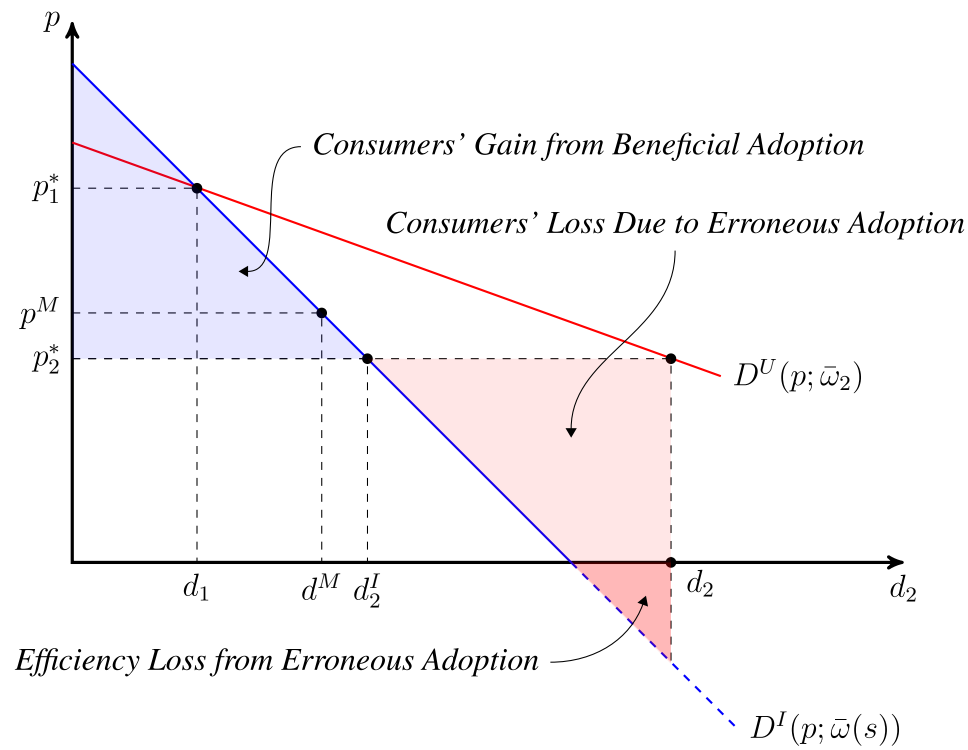
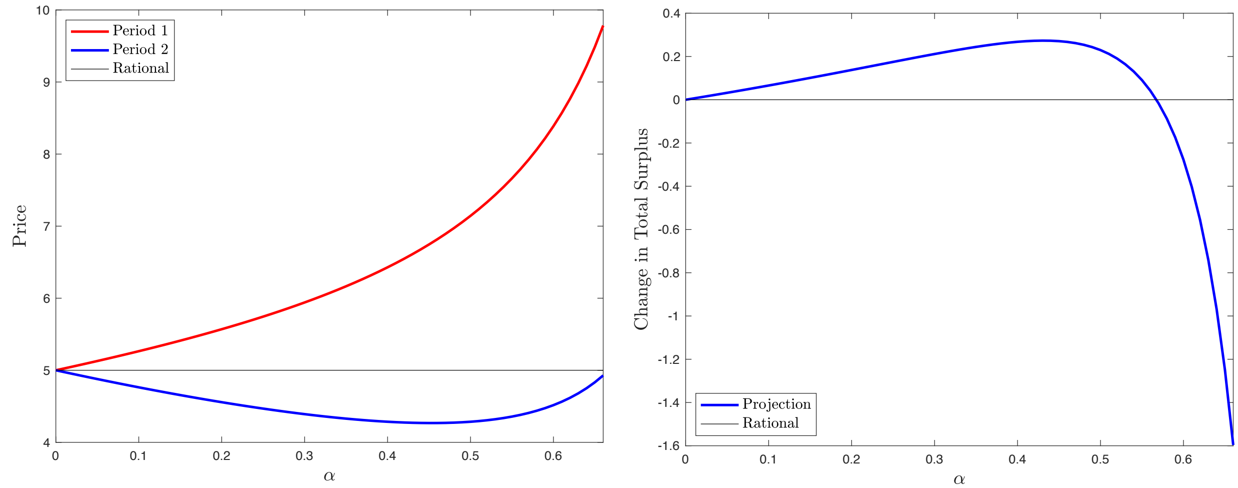


Figure 3: *Demand functions in period 2 (for both informed and uninformed agents).*

exhibit near-rational levels of variation, so a reduction in p_2 will not attract many more buyers than it would under rational learning. Hence, there is little benefit in deviating from the rational monopoly price. But as α increases, perceived valuations become more clustered around $\bar{\omega}_2$, meaning that a price drop will attract a bigger proportion of the market and will thus be more profitable. This explains why p_2^* initially decreases in α. However, once α is sufficiently large—and thus beliefs are substantially inflated due to a high p_1^*—the seller can capture a significant fraction of the market with a smaller deviation from p^M.

While harming uninformed consumers, projection can actually increase total surplus when the bias is not too severe. Indeed, total quantity demanded across both periods can be higher under projection than the rational benchmark. This reduces the traditional deadweight loss due to monopoly pricing. However, this inflated level of sales can sometimes be detrimental to total surplus, since sufficiently strong projection can induce inefficient over-consumption (as in Proposition 5). Panel (b) of Figure 4 shows how total surplus changes as a function of α; total surplus begins to fall once sales have expanded to the point where those with negative valuations are lured into buying.



(a) Optimal Prices (p_n^*)

(b) Change in Total Surplus

Figure 4: *Optimal prices and the effect of projection on total surplus as a function of* α

4.3 Beyond Two Periods

We now briefly consider how biased learning unfolds for longer horizons. Notably, we show for an arbitrary number of periods that consumers' beliefs continue to obey the central comparative statics identified above: expectations of ω are decreasing in private tastes and increasing in past prices.

Inference among uninformed agents in any period $n > 2$ is similar to inference in the two-period model with one marked difference. While the inferential error of consumers in Generation 2 stems directly from misunderstanding others' tastes (i.e., an error in first-order beliefs), the mistaken inference in later generations also includes a "social misinference" effect stemming from naivete about others' projection: individuals neglect that their predecessors failed to reach consistent beliefs. Indeed, because uninformed consumers expect to extract s form their predecessors' behavior, an individual in period n accordingly thinks that the uninformed consumers in period $n-1$ consistently and correctly inferred s. This presumption is false since projectors in period $n-1$ formed biased, taste-dependent beliefs. Nevertheless, an uninformed projector in Generation n with taste t thinks the observed demand from Generation $n-1$ is again determined by the (misperceived) informed demand function $\widehat{D}^I(p_{n-1}; \hat{\omega}|t)$ from (9)—she does not realize that it derives from a composition of demand functions as in (12). This observer then infers a value of $\hat{\omega}$ that solves $d_{n-1} = \widehat{D}^I(p_{n-1}; \hat{\omega}|t)$, which we denote by $\hat{\omega}_n(t)$. Despite these additional complexities, our next proposition shows that the key features of beliefs identified in the two-period model extend to any number of periods.

Proposition 6. *Suppose* (p_n, s) *admits interior demand for all* $n = 1, \ldots, N$*, and consider any* $\alpha > 0$*. For each period* $n = 2, \ldots, N$ *and each type* $t \in \mathcal{T}$*:*

1. $\hat{\omega}_n(t)$ *can be written as* $\hat{\omega}_n(t) = \bar{\omega}_n - \alpha t$*, where* $\bar{\omega}_n$ *is independent of* t*.*

2. *$\bar{\omega}_n$ is strictly increasing in each p_k for $k < n$.*

Part 1 of Proposition 6 implies that, in every period, a projector's inferred quality is decreasing in her private taste. Moreover, since the taste-independent component of beliefs in any period is a sufficient statistic for a specific type's belief, the evolution of beliefs of *all* types can be simply summarized by the unidimensional process $(\bar{\omega}_n)$.

Part 2 of Proposition 6 shows that the biased perceptions in a given period n are increasing in all past prices; hence, the manipulating effect of prices on the beliefs of projectors is amplified compared to the static setting of Section 3. The proof further illustrates how the beliefs derived in the static model correspond to the steady state of this dynamic model: if prices were held constant, then $\bar{\omega}_n = \bar{\omega}_2$ for all $n > 2$ and thus $\hat{\omega}_n(t)$ matches the static belief in (8). Proposition 6 also implies that prices in earlier periods have a long-lasting effect on beliefs. Relative to our analysis of optimal monopoly pricing in the two-period model, this result adds a further incentive for the monopolist to use a declining-price strategy. Online Appendix C shows that indeed such a strategy is optimal for any arbitrary horizon when tastes are uniformly distributed, and we conjecture that this result further generalizes given that the evolution of projectors' beliefs displays the same properties as in the two-period model. Intuitively, the seller balances a trade-off between manipulating the beliefs of future consumers by maintaining a high current price versus exploiting consumers' current beliefs by undercutting the previous price.[30]

5 Further Applications and Extensions

In this section, we discuss further implications of taste projection when we relax our assumptions that consumers (i) are short lived and (ii) have unit demand.

5.1 Endogenous Timing and Under-appreciation of Selection Effects

Section 4 showed how high-to-low pricing induces "short-lived" low-valuation projectors to excessively adopt the good. We now show that such over-adoption arises even if the price is fixed when consumers are "long-lived" and choose when to buy. Thus, the idea that projection causes consumers to be overly influenced by earlier purchases is not limited to settings with changing prices.

Consider a variant of our dynamic model from Section 4 where, instead of a new mass of consumers entering in the second period, there is a single group of consumers with unit demand who can buy in either period (or not at all). We assume that the price p is fixed across periods. We additionally assume T is uniform to ease exposition, but the logic will transparently generalize. Finally,

[30] In this way, our model predicts a gradual decline in prices, which is consistent with the pattern observed for novel products with uncertain quality; see Bayus (1992), Krishnan et al. (1999), and Liu (2010).

a fraction $\lambda \in (0, 1)$ of consumers observe s while $1 - \lambda$ are uninformed.

Informed agents buy in period 1 or never, since they have nothing to learn from delaying; they buy immediately if $\bar{\omega}(s) + t \geq p$.[31] Uninformed agents with low private values may defer their purchase decision to period 2 in order to learn from those adopting in period 1. Specifically, an uninformed agent buys in period 1 if $\bar{\omega}_0 + t \geq p$, where $\bar{\omega}_0$ reflects the expected quality among uninformed agents.[32] Otherwise, they observe the quantity demanded in period 1, form an updated expectation $\hat{\omega}_2(t)$, and then buy in period 2 if $\hat{\omega}_2(t) + t \geq p$.

The quantity demanded in period 1 is $d_1 = \lambda D(p; \bar{\omega}(s)) + (1 - \lambda) D(p; \bar{\omega}_0)$, where $D(p; \omega) = 1 - F(p - \omega)$. As usual, a projecting agent in period 2 with taste t updates their belief to $\hat{\omega}_2(t)$, which is the value $\hat{\omega}$ that fits their model to the observed outcome: $\hat{\omega}$ solves $d_1 = \lambda \widehat{D}(p; \hat{\omega}|t) + (1 - \lambda)\widehat{D}(p; \bar{\omega}_0|t)$, where $\widehat{D}(p; \omega|t) = 1 - \widehat{F}(p - \omega|t)$. To state our next result, we impose some convenient technical assumptions ensuring that under both rational and biased inference there are well-defined marginal types in period 2, which we denote by t_2^* and $\hat{t}_2$, respectively. Namely, suppose that $D(p; \bar{\omega}_0) \in (0, 1)$, $\widehat{D}(p; \bar{\omega}_0|\underline{t}) > 0$, and $d_1 \leq \lambda + (1 - \lambda)\widehat{D}(p; \bar{\omega}_0|\underline{t})$. The first condition means that an interior fraction of uninformed agents delay. The final two conditions mean that all projectors expect an interior fraction to delay and the observed demand is consistent with their models; this happens when λ is sufficiently large compared to α.

Proposition 7. *Suppose (p, s) admits interior demand and $\lambda > \alpha > 0$.*

1. *Suppose informed agents have positive information about the good; i.e., $\bar{\omega}(s) > \bar{\omega}_0$. (i) The quantity demanded in period 2 exceeds the rational benchmark, and the range of types who suboptimally adopt, $\left[\hat{t}_2, t_2^*\right]$, is increasing in both α and $\bar{\omega}(s) - \bar{\omega}_0$. (ii) There exists a threshold value $\tilde{t} > t_2^*$ such that all types $t \in \left[\hat{t}_2, \tilde{t}\right]$ will, on average, receive lower quality than they expect; i.e., $t < \tilde{t}$ implies $\mathbb{E}[\omega - \hat{\omega}_2(t)|s] < 0$.*

2. *Suppose informed agents have negative information about the good; i.e., $\bar{\omega}(s) < \bar{\omega}_0$. Then there is zero demand in period 2, as in the rational benchmark.*

Proposition 7 stems from projectors underestimating a selection effect that naturally emerges in this environment: consumers who decide to buy in period 1 tend to have higher private values than those who delay. Those who delay are aware of this selection effect, but they underestimate it. Since the delayers systematically underestimate the private values of those who buy early, they over-attribute observations from period 1 to quality rather than differences in tastes. When d_1 is stronger than expected, delayers become too optimistic and too many of them buy—they are subsequently disappointed by the quality they receive. When d_1 is weaker than expected, delayers become too

[31]This relies on a mild (implicit) assumption that delaying consumption is costly to consumers.

[32]Our conclusions in this application would not change if $\bar{\omega}_0$ were to depend on p—which might naturally occur if p partially signals quality—so long as informed consumers have additional information that is not revealed by p.

pessimistic and don't buy. However, they would not buy based on this bad news even under rational learning: since they were unwilling to buy with belief $\bar{\omega}_0$, they are only willing to buy in period 2 if they receive good news. Hence, projection generates an asymmetric bias in behavior, leading to over-adoption among delayers, but not under-adoption.

Additionally, insofar as unmet quality expectations drive negative product reviews, the fact that over-adoption is coupled with systematic disappointment may explain why high initial reviews for a product are too frequently followed by negative reviews (Li and Hitt, 2008; Papanastasiou et al., 2015; Dai et al., 2018). Taste projection therefore provides a potential mechanism underlying the sort of selection neglect suggested in this literature on learning from reviews.

5.2 Multi-Unit Demand

We now revisit the static equilibrium from Section 3 but allow consumers to have multi-unit demand. As before, consumers still form type-dependent beliefs that are negatively related to their tastes. In contrast to that previous case, however, projectors now fine-tune their actions to their erroneous beliefs. Thus, all projecting types generically consume a sub-optimal amount in equilibrium, leading to potentially large inefficiencies. In particular, since perceptions are negatively related to tastes, high types underconsume while low types overconsume.[33]

For simplicity, we consider the familiar case of quadratic utility (e.g., Judd and Riordan, 1994; Caminal and Vives, 1996), where a consumer's valuation for x units of the good is given by $u(x;\omega,t) = (\omega+t)x - x^2/2$. A consumer with a quality expectation of $\hat{\omega}$ facing a per-unit price of p then demands a quantity $x^*(p;\hat{\omega},t) = \hat{\omega}+t-p$ if $\hat{\omega}+t-p \geq 0$ and $x^*(p;\hat{\omega},t) = 0$ otherwise.

As in Section 3, a fraction $\lambda \in (0,1)$ of consumers observe s and form a quality expectation $\bar{\omega}(s)$. The remaining fraction $1-\lambda$ form this expectation based on the aggregate demand at price p. The steady-state equilibrium is analogous to the one defined above: uninformed agents make inferences that are consistent with the observed quantity demanded and their misspecified model, and the resulting quantity is consistent with those beliefs. More specifically, let $\hat{\omega}(t)$ be type t's quality expectation in equilibrium. Aggregate demand in equilibrium is thus

$$d = \lambda \cdot \underbrace{\int_{\mathcal{T}} x^*\big(p;\bar{\omega}(s),t\big)dF(t)}_{\text{Informed Demand}} + (1-\lambda) \cdot \underbrace{\int_{\mathcal{T}} x^*\big(p;\hat{\omega}(t),t\big)dF(t)}_{\text{Uninformed Demand}}. \tag{15}$$

Since uninformed agents expect that all types reach a common and correct expectation of ω in equilibrium, each $\hat{\omega}(t)$ is the value that predicts quantity d under type t's model given the presumption

[33] Although this is intuitive given our earlier results on biased perceptions, it is nevertheless important to verify that projection induces steady-state inefficiencies. The reason such inefficiencies were absent in Section 3 was merely an artifact of the unit-demand structure.

that all types have inferred this same value.

Proposition 8. *Suppose (p, s) admits positive aggregate demand among informed consumers. For any $\alpha > 0$, there exists a unique equilibrium profile of beliefs $\hat{\omega}(t)$ with the following properties:*

1. *Quality perceptions are negatively related to tastes: $\hat{\omega}(t)$ is strictly decreasing in t.*

2. *Relative to the rational benchmark, high types demand too little and low types demand too much: there exists an interior threshold type $\tilde{t}$ such that $t > \tilde{t}$ implies $x^*\big(p; \hat{\omega}(t), t\big) < x^*\big(p; \bar{\omega}(s), t\big)$ and $t < \tilde{t}$ implies $x^*\big(p; \hat{\omega}(t), t\big) > x^*\big(p; \bar{\omega}(s), t\big)$.*

3. *Relative to the rational benchmark, demand along the extensive margin increases: the lowest uninformed type who buys a positive quantity is lower than the lowest type who buys a positive quantity in the rational benchmark.*

4. *More extreme types exhibit greater inefficiency: $\big|x^*\big(p; \hat{\omega}(t), t\big) - x^*\big(p; \bar{\omega}(s), t\big)\big|$ is strictly increasing in $|t - \tilde{t}|$.*

The intuition for Part 1 of Proposition 8 is identical to the unit-demand case. However, consumers now tailor their individual demand to their idiosyncratic beliefs. This underlies Part 2: since high types are typically pessimistic about the product's quality, they consume too little; low types instead consume too much. In this sense, consumption along the intensive margin is reduced, since projection reduces the quantity demanded among the high types who consume the most. But consumption along the *extensive* margin increases (Part 3). That is, the set of types who consume the good in equilibrium expands: some low types who would entirely abstain under rational inference are now persuaded to buy the product. Parts 2 and 3 together imply that, relative to the rational benchmark, consumption is spread more thinly across a wider range of buyers.

The logic behind these results is quite transparent as $\alpha \to 1$. In this case, projectors think there is essentially no heterogeneity in tastes, and that aggregate demand derives from all individuals consuming roughly the same quantity. From a projector's point of view, the average quantity demanded is then a near perfect signal about how much he himself should consume—he should consume that same amount, since he is just like everybody else. Thus, in equilibrium, the difference in consumption across types narrows, while the set of types who consume expands.

Finally, among the segment of consumers who buy in equilibrium, those with types closer to the extremes make worse decisions (Part 4). Intuitively, these types are farther from the average buyer, and thus their mental model provides a worse interpretation of the data. A truly average projecting consumer is fairly accurate when she imagines that most people share her tastes. But those with more extreme tastes hold a more distorted model of the world by assuming their tastes are typical. Proposition 8, along with the results of Section 4, reveal that where the burden of projection falls depends on the demand structure: with single-unit demand, it is only low types who can be manipulated into inefficiently adopting a product; with multi-unit demand, the burden falls on extreme types, either high or low.

6 Discussion and Conclusion

Evidence suggests that people often misperceive others' tastes, attitudes, and motives by exaggerating the similarity between others and themselves. In this paper, we have examined a model where consumers interpret market data through the lens of these misperceptions. In contexts where consumers aim to learn the commonly-valued quality of a product from others' demand, we showed that projection leads to systematically distorted beliefs. Namely, projecting consumers will form quality estimates that are decreasing in their tastes and increasing in the product's price. These misinferences create new pricing incentives for a monopolist: in a dynamic setting, the seller will charge a high initial price to inflate future consumers' beliefs and then will lower the price to capitalize on these distorted beliefs. Projection also has implications for efficiency. For instance, both dynamic pricing and a failure to appreciate selection effects can lead projectors to inefficiently over-adopt a good.[34] We conclude with two lines of discussion. First, we note how our main results are robust to generalizations of our model. Second, we discuss other potential applications of our model.

In order to streamline the analysis, we posited that a projector's misperception of others' tastes is a convex combination of his own and others' tastes, which implies that his perceived distribution of types is a counter-clockwise rotation of the true distribution around his own type. Yet, our misinference results are more general. In fact, directly assuming this rotation property would also deliver many of our results.[35] Our misinference results also extend to more general utility functions (see the proofs of Propositions 1 and 2). Furthermore, for ease of exposition, we focused on a simple social-learning environment where all informed agents observe the same signal. In Online Appendix B, we show that our main comparative statics continue to hold under two richer information structures: (i) "fully heterogeneous signals," where each agent observes an independent private signal; (ii) "heterogeneous signals across periods," where all agents within each period n observe a common signal that is unobserved by agents in other periods. Similarly, while we have assumed that consumers' private values are independently distributed, our model of taste projection can also accommodate the case of correlated private values; see Gagnon-Bartsch et al. (2021a).

There are several other potential applications of our framework. As discussed above, projection

[34] Note that our statements about efficiency implicitly disregard externatlities. Projection could be beneficial from a social-welfare perspective when large-scale adoption is a critical objective (e.g., adoption of clean-energy technologies).

[35] In a similar vein, a *clockwise* rotation of F would capture an opposing bias whereby a consumer thinks others' tastes differ from his by more than they actually do. This assumption would reverse our results that stem from consumers exaggerating the elasticity of demand; namely, it would imply quality perceptions that decrease in observed prices.

leads to lower dispersion in consumers' valuations and hence to a counter-clockwise rotation of the market demand curve. The insights from Johnson and Myatt (2006) should therefore apply to a market with projectors. For instance, in a setting where a monopolist engages in second-degree price discrimination by offering a menu of multi-unit bundles, they show that a counter-clockwise rotation of the demand curve can lead the seller to prefer a smaller menu. Thus, a seller should have a similar preference when facing projecting consumers.

A natural extension of our framework would be to study the role of projection in markets with multiple sellers. Consider a setting with a "dominant" firm and a competitive fringe of smaller price-taking firms. Let the dominant firm act as the monopolist from our model above, while the fringe firms supply a product of known quality ω_f at price $c \geq 0$. Furthermore, suppose that the fringe product is inferior to the one supplied by the dominant firm and that all firms make positive sales.[36] In this extension, a projector's belief about the quality of the dominant firm's product would still increase in its price and decrease in her own private taste. Moreover, projectors become more optimistic about the quality of the dominant firm's product the larger is the surplus they could obtain from the fringe, $(\omega_f - c)$. Intuitively, for a projector who is trying to learn the quality of the dominant firm's product from its demand, a given sales volume provides a more positive signal when the fringe product is more appealing. Thus, we suspect that the misinference due to projection that we have identified for a monopoly would still arise in some competitive settings.

Finally, projection may also distort an individual's perception of her information sources. Suppose consumers entertain the possibility that others are biased in favor of a particular option (e.g., a particular brand, author, or politician), supporting it even when they know it has low quality. Even when such blind support is absent in reality, projectors are prone to think it exists. This is because a projector who tries the product and learns its true quality will then observe a puzzling degree of popularity. For instance, a projector who despises an option will see far too many people (in her eyes) supporting it. To make sense of this discrepancy, she may come to believe that others have some ulterior motive, since she neglects that their support could come from mere differences in taste. Such skepticism of others' motives may lead people to discredit others' actions, shedding light on why some groups are unmoved by others' actions even when they reveal valuable information.

[36] Specifically, consider a situation where consumers have weakly positive private values, and a consumer with taste t values the fringe's product at γt, with $\gamma \in (0, 1)$.

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Appendix

A Proofs

Proof of Proposition 1. We prove this result for more general preferences than assumed in the main text. Here, we assume that each agent's valuation is given by a utility function $u(\omega, t)$ that is strictly increasing and differentiable with respect to both variables and satisfies $\frac{\partial^2}{\partial\omega\partial t}u(\omega, t) \geq 0$ for all $t \in \mathcal{T}$ and $\omega \in \mathbb{R}$. For simplicity, we also assume u is linear in ω.[37] Our model of projection easily accommodates such a generalization: An agent with private value t believes the utility of any agent with taste t' is $\hat{u}(\omega, t'|t) = \alpha u(\omega, t) + (1 - \alpha)u(\omega, t')$. This misperceived utility function then pins down type t's perceived distribution of valuations in each state ω. We begin by proving the following lemma.

Lemma A.1. *Consider any u satisfying the assumptions above, and suppose that (p, s) admits interior demand. For any $\lambda > 0$ and $\alpha \in [0, 1)$, there exists a unique steady-state equilibrium; in that equilibrium, the quantity demanded is equal to the quantity demanded in the full-information benchmark (i.e., $\lambda = 1$).*

Step 1: Inference rules. We first derive an uninformed agent's inference from the observed quantity demanded, d. Since we focus on symmetric strategies, it is sufficient to derive the inference rule of an arbitrary agent with taste t. Let $\widehat{D}(p; \hat{\omega}|t)$ denote this agent's conjectured demand among a population of agents who believe the expected value of ω is $\hat{\omega}$;

$$\begin{aligned}\widehat{D}(p; \hat{\omega}|t) = \Pr\Big[\alpha u(\hat{\omega}, t) + (1 - \alpha)u(\hat{\omega}, T) \geq p\Big] &= \Pr\left[u(\hat{\omega}, T) \geq \frac{p - \alpha u(\hat{\omega}, t)}{1 - \alpha}\right] \\ = \Pr\left[T \geq t^*\left(\frac{p - \alpha u(\hat{\omega}, t)}{1 - \alpha}; \hat{\omega}\right)\right] &= 1 - F\left(t^*\left(\frac{p - \alpha u(\hat{\omega}, t)}{1 - \alpha}; \hat{\omega}\right)\right), \end{aligned} \quad \text{(A.1)}$$

where $t^*(p; \hat{\omega})$ is the inverse of $u(\hat{\omega}, t)$ w.r.t. t evaluated at $\hat{\omega}$ and p. That is, $t^*(p; \hat{\omega})$ is such that $u(\hat{\omega}, t^*(p; \hat{\omega})) = p$ for all $p \geq 0$ and $\hat{\omega} \in \mathbb{R}$. Note that t^* is well defined given our assumptions on u. Furthermore, let $t_1^*(p; \hat{\omega})$ and $t_2^*(p; \hat{\omega})$ denote the partial derivative of t^* w.r.t. the first and second argument, respectively; our assumptions on u also imply that for all $p \geq 0$ and $\hat{\omega} \in \mathbb{R}$, we have $t_1^*(p; \hat{\omega}) > 0$ and $t_2^*(p; \hat{\omega}) < 0$.

The inference rule of an uninformed agent with taste t is then given by the function $\hat{\omega}(\cdot|t, p) : [0, 1] \to \mathbb{R}$ such that for all $d \in (0, 1)$, $\hat{\omega}(d|t, p)$ is equal to the unique value of $\hat{\omega}$ that solves

$$d = 1 - F\left(t^*\left(\frac{p - \alpha u(\hat{\omega}, t)}{1 - \alpha}; \hat{\omega}\right)\right), \quad \text{(A.2)}$$

[37] The intuitions from the proof generalize beyond this risk-neutral case. However, we assume risk neutrality so that, as in the main text, each agent's mean belief, $\hat{\omega}$, is a sufficient statistic for their behavior irrespective of further details on their posterior distribution over ω. Thus, as in the main text, uninformed agents here attempt to extract the mean belief of informed agents, $\bar{\omega}(s)$. The proof below holds without the linearity assumption when informed agents are perfectly informed. And an analogous argument would hold beyond the linear case so long as we impose a similar structure on an agent's *expected* utility conditional on s and t.

and $\hat{\omega}(d|t,p)$ represents the agent's perceived expected value of ω. An uninformed agent with taste t buys if d is such that $u\left(\hat{\omega}(d|t,p),t\right) \geq p$. The steady-state equilibrium condition is then:

$$d = \lambda D^I(p;\bar{\omega}(s)) + (1-\lambda)\Pr\left[u\left(\hat{\omega}(d|T,p),T\right) \geq p\right], \tag{A.3}$$

where $\Pr\left[u\left(\hat{\omega}(d|T,p),T\right) \geq p\right]$ is the true fraction of uninformed agents who buy in the steady state.

Under our solution concept, a projecting agent with taste t believes that all agents (i) follow the same inference rule as him; (ii) form an expectation of ω equal to $\hat{\omega}(d|t,p)$; and (iii) take their expected-utility-maximizing action given this expectation. He therefore believes that, in equilibrium, his inference rule allows him to perfectly extract the signal of the informed agents. To see this, note that an agent with taste t thinks that demand among the informed is

$$\widehat{D}(p;\bar{\omega}(s)|t) = 1 - F\left(t^*\left(\frac{p-\alpha u(\bar{\omega}(s),t)}{1-\alpha};\bar{\omega}(s)\right)\right), \tag{A.4}$$

and thinks that

$$\begin{aligned}\Pr\left[u\left(\hat{\omega}(d|T,p),T\right) \geq p\right] &= \Pr\left[u\left(\hat{\omega}(d|t,p),T\right) \geq p\right] \\ &= 1 - F\left(t^*\left(\frac{p-\alpha u(\hat{\omega}(d|t,p),t)}{1-\alpha};\hat{\omega}(d|t,p)\right)\right) = d\end{aligned} \tag{A.5}$$

where the first equality follows because a projector with type t believes that all types infer the same value as himself, $\hat{\omega}(d|t,p)$, and the third equality follows from the fact that, by definition, $\hat{\omega}(d|t,p)$ is the value of $\hat{\omega}$ that solves (A.2). Thus, substituting (A.4) and (A.5) into (A.3) reveals that the agent believes that, in equilibrium, the aggregate quantity demanded is such that

$$\begin{aligned}&d = \lambda\left(1 - F\left(t^*\left(\frac{p-\alpha u(\bar{\omega}(s),t)}{1-\alpha};\bar{\omega}(s)\right)\right)\right) + (1-\lambda)d \\ &\qquad\qquad\Rightarrow\quad d = 1 - F\left(t^*\left(\frac{p-\alpha u(\bar{\omega}(s),t)}{1-\alpha};\bar{\omega}(s)\right)\right).\end{aligned} \tag{A.6}$$

Within this agent's model, both (A.5) and (A.6) must hold, and hence the agent believes

$$1 - F\left(t^*\left(\frac{p-\alpha u(\bar{\omega}(s),t)}{1-\alpha};\bar{\omega}(s)\right)\right) = 1 - F\left(t^*\left(\frac{p-\alpha u(\hat{\omega}(d|t,p),t)}{1-\alpha};\hat{\omega}(d|t,p)\right)\right), \tag{A.7}$$

which implies that $\hat{\omega}(d|t,p) = \bar{\omega}(s)$ since $\hat{\omega}(d|t,p)$ is the unique value of $\hat{\omega}$ that solves (A.2). Thus, the projector's inference rule would correctly identify $\bar{\omega}(s)$ *if* his model were correct (but it's not).

By this logic, this inference rule does correctly reveal the informed agents' private information when all agents are rational (i.e., $\alpha = 0$), since in this case (A.7) reduces to $t^*(p;\bar{\omega}(s)) = t^*(p;\hat{\omega}(d|t,p))$ and thus in reality we have $\hat{\omega}(d|t,p) = \bar{\omega}(s)$ since t^* is strictly decreasing in $\hat{\omega}$.

Step 2: $\hat{\omega}(d|t,p)$ is strictly decreasing in t. Next, we show that $\hat{\omega}(d|t,p)$ is strictly decreasing in t. Recall that for any fixed $d \in (0,1)$, Condition (A.2) implies that $\hat{\omega}(d|t,p)$ solves

$$L(\hat{\omega}|t,p) \equiv t^*\left(\frac{p-\alpha u(\hat{\omega},t)}{1-\alpha};\hat{\omega}\right) - F^{-1}(1-d) = 0. \tag{A.8}$$

By the Implicit Function Theorem, we have

$$\frac{\partial \hat{\omega}(d|t,p)}{\partial t} = -\left(\frac{\partial L(\hat{\omega}|t,p)}{\partial t}\right)\left(\frac{\partial L(\hat{\omega}|t,p)}{\partial \hat{\omega}}\right)^{-1}\Bigg|_{\hat{\omega}=\hat{\omega}(d|t,p)}. \tag{A.9}$$

Notice that

$$\frac{\partial L(\hat{\omega}|t,p)}{\partial t} = -t_1^*\!\left(\frac{p-\alpha u(\hat{\omega},t)}{1-\alpha};\hat{\omega}\right)\left(\frac{\alpha}{1-\alpha}\right)\frac{\partial u(\hat{\omega},t)}{\partial t} < 0, \tag{A.10}$$

and

$$\frac{\partial L(\hat{\omega}|t,p)}{\partial \hat{\omega}} = -t_1^*\!\left(\frac{p-\alpha u(\hat{\omega},t)}{1-\alpha};\hat{\omega}\right)\left(\frac{\alpha}{1-\alpha}\right)\frac{\partial u(\hat{\omega},t)}{\partial \hat{\omega}} + t_2^*\!\left(\frac{p-\alpha u(\hat{\omega},t)}{1-\alpha};\hat{\omega}\right) < 0, \tag{A.11}$$

and hence (A.9) implies that $\frac{\partial \hat{\omega}(d|t,p)}{\partial t} < 0$.

Step 3: Total perceived valuations, $u(\hat{\omega}(d|t,p),t)$, are increasing in t. Although perceived quality is decreasing in t (Step 2), total perceived valuations remain increasing in t. Notice that

$$\frac{du(\hat{\omega}(d|t,p),t)}{dt} = \frac{\partial u(\hat{\omega}(d|t,p),t)}{\partial \hat{\omega}}\frac{\partial \hat{\omega}(d|t,p)}{\partial t} + \frac{\partial u(\hat{\omega}(d|t,p),t)}{\partial t}, \tag{A.12}$$

and thus $\frac{du(\hat{\omega}(d|t,p),t)}{dt} > 0$ if

$$\frac{\partial \hat{\omega}(d|t,p)}{\partial t} > -\left(\frac{\partial u(\hat{\omega},t)}{\partial t}\right)\left(\frac{\partial u(\hat{\omega},t)}{\partial \hat{\omega}}\right)^{-1}\Bigg|_{\hat{\omega}=\hat{\omega}(d|t,p)}. \tag{A.13}$$

Substituting (A.10) and (A.11) into (A.9) implies that

$$\frac{\partial \hat{\omega}(d|t,p)}{\partial t} = -\left(\frac{\partial u(\hat{\omega},t)}{\partial t}\right)\left(\frac{\partial u(\hat{\omega},t)}{\partial \hat{\omega}} + K\right)^{-1}\Bigg|_{\hat{\omega}=\hat{\omega}(d|t,p)}, \tag{A.14}$$

where

$$K = -\left(\frac{1-\alpha}{\alpha}\right)\underbrace{t_2^*\!\left(\frac{p-\alpha u(\hat{\omega},t)}{1-\alpha};\hat{\omega}\right)}_{<0}\underbrace{\left(t_1^*\!\left(\frac{p-\alpha u(\hat{\omega},t)}{1-\alpha};\hat{\omega}\right)\right)^{-1}}_{>0}\Bigg|_{\hat{\omega}=\hat{\omega}(d|t,p)}, \tag{A.15}$$

and hence (A.13) holds given that $K \geq 0$. Note that K is strictly positive if $\alpha > 0$ and hence equilibrium total perceived valuations are strictly increasing in t under projection.

Step 4: The fraction of uninformed agents who buy follows a cutoff rule and is equal to fraction of informed agents who buy. The equilibrium condition in (A.3) depends on the fraction of uninformed agents who buy in the steady state, $\Pr\left[u\left(\hat{\omega}(d|T,p),T\right) \geq p\right]$. Since Step 3 ensures that $u\left(\hat{\omega}(d|t,p),t\right)$ is strictly increasing in t, there must exist a threshold value $\hat{t}(d)$ such that, in equilibrium, types with with $t \geq \hat{t}(d)$ buy and those with $t < \hat{t}(d)$ do not. That is, there is a well-defined "marginal uninformed type", $\hat{t}(d)$, that separates the type space into buyers and non-buyers.

We now show that, for any value of $d \in (0,1)$, it must be that $\hat{t}(d) = F^{-1}(1-d)$. That is, the marginal uninformed type is such that the fraction of uninformed agents who buy is equal to d. To see this, the inference of an agent of any type t, $\hat{\omega}(d|t,p)$, must satisfy

$$u\left(\hat{\omega}(d|t,p), t^*\left(\frac{p-\alpha u(\hat{\omega}(d|t,p),t)}{1-\alpha};\hat{\omega}(d|t,p)\right)\right) = \frac{p-\alpha u(\hat{\omega}(d|t,p),t)}{1-\alpha}; \tag{A.16}$$

this follows from the fact that, by definition, $t^*(\tilde{u};\hat{\omega}(d|t,p))$ is the value of t such that $u(\hat{\omega}(d|t,p),t) = \tilde{u}$. Furthermore, recall that for all t, the inference rule $\hat{\omega}(d|t,p)$ is such that (A.8) holds as an identity; substituting this identity into (A.16) and rearranging implies that

$$p = \alpha u(\hat{\omega}(d|t,p),t) + (1-\alpha)u\big(\hat{\omega}(d|t,p), F^{-1}(1-d)\big). \tag{A.17}$$

Given that the condition above must hold for all $t \in \mathcal{T}$, it must hold for type $\hat{t}(d) \equiv F^{-1}(1-d)$ whose private value lies at the $(1-d)$-percentile in the taste distribution. Condition (A.17) evaluated at $\hat{t}(d) = F^{-1}(1-d)$ implies

$$\begin{aligned} p &= \alpha u\big(\hat{\omega}(d|\hat{t}(d),p), F^{-1}(1-d)\big) + (1-\alpha)u\big(\hat{\omega}(d|\hat{t}(d),p), F^{-1}(1-d)\big) \\ &= u\big(\hat{\omega}(d|\hat{t}(d),p), \hat{t}(d)\big). \end{aligned} \tag{A.18}$$

Thus, an agent with type $\hat{t}(d) = F^{-1}(1-d)$ forms an inference that leaves him indifferent between buying or not. By Step 3, above, an agent with $t > \hat{t}(d)$ must form an inference yielding a strict preference to buy, while one with $t < \hat{t}(d)$ must form an inference yielding a strict preference to not buy. Thus $\hat{t}(d)$ represents the marginal uninformed type, and the fraction of uninformed agents who buy is thus $\Pr\left[u\left(\hat{\omega}(d|T,p),T\right) \geq p\right] = 1 - F\left(\hat{t}(d)\right) = 1 - F(F^{-1}(1-d)) = d$.

Recall from (A.3) that, in equilibrium, the aggregate quantity demanded must satisfy

$$d = \lambda D^I(p;\bar{\omega}(s)) + (1-\lambda)\Pr\left[u\left(\hat{\omega}(d|T,p),T\right) \geq p\right]. \tag{A.19}$$

From above, $\Pr\left[u\left(\hat{\omega}(d|T,p),T\right) \geq p\right] = d$, and hence the equilibrium condition reduces to

$$d = \lambda D^I(p;\bar{\omega}(s)) + (1-\lambda)d \quad \Rightarrow \quad d = D^I(p;\bar{\omega}(s)). \tag{A.20}$$

This completes the proof of the lemma. We now establish each part of Proposition 1.

Part 1. Let $\hat{\omega}(t)$ denote the steady-state inference of an uninformed agent who has taste t; that is, $\hat{\omega}(t) \equiv \hat{\omega}(d^*|t,p)$, where $d^* \equiv D^I(p;\bar{\omega}(s))$ is the quantity demanded in equilibrium. The fact that $\hat{\omega}(t)$ is strictly decreasing in t is established in Step 2 in the proof of Lemma A.1.

Recall from Step 4 of Lemma A.1 that the marginal uninformed type is $\hat{t}(d) = F^{-1}(1-d)$. Since $d = D^I(p;\bar{\omega}(s)) = [1 - F(t^*(p;\bar{\omega}(s)))]$ in equilibrium, we therefore have $\hat{t}(d) = t^*(p;\bar{\omega}(s))$ in equilibrium. That is, the marginal uninformed type is equal to the marginal informed type. This further implies that an uninformed agent with $t = t^*(p;\bar{\omega}(s))$ is the unique uninformed type who correctly infers s: substituting $\hat{t}(d) = t^*(p;\bar{\omega}(s))$ into (A.18) implies that this type forms an inference that leaves him indifferent between buying or not, which means that he must form the same expectation as the informed agent who is truly indifferent; hence, $\hat{\omega}(d|t^*(p;\bar{\omega}(s)),p) = \bar{\omega}(s)$ at the equilibrium value of d. Since $\hat{\omega}(t)$ is strictly decreasing in t, this implies that uninformed agents with $t > t^*(p;\bar{\omega}(s))$ underestimate the state, while those with $t < t^*(p;\bar{\omega}(s))$ overestimate the state.

Part 2. We now argue that $\hat{\omega}(t)$ is increasing in p for each $t \in \mathcal{T}$. Condition (A.2) implies that

$\hat{\omega}(d|t,p)$ solves

$$L(\hat{\omega}|t,p) \equiv t^*\left(\frac{p-\alpha u(\hat{\omega},t)}{1-\alpha};\hat{\omega}\right) - F^{-1}(1-d) = 0. \tag{A.21}$$

In the steady-state, $d = D^I(p;\bar{\omega}(s)) = 1 - F(t^*(p;\bar{\omega}(s)))$ and hence $F^{-1}(1-d) = t^*(p;\bar{\omega}(s))$; the preceding condition implies that $\hat{\omega}(t)$ solves

$$L(\hat{\omega}|t,p) \equiv t^*\left(\frac{p-\alpha u(\hat{\omega},t)}{1-\alpha};\hat{\omega}\right) - t^*(p;\bar{\omega}(s)) = 0. \tag{A.22}$$

The Implicit Function Theorem then implies $\frac{\partial\hat{\omega}(t)}{\partial p} = -\left(\frac{\partial L(\hat{\omega}|t,p)}{\partial p}\right)\left(\frac{\partial L(\hat{\omega}|t,p)}{\partial \hat{\omega}}\right)^{-1}\Big|_{\hat{\omega}=\hat{\omega}(t)}$, and (A.11) shows that $\frac{\partial L(\hat{\omega}|t,p)}{\partial\hat{\omega}} < 0$. Hence, $\frac{\partial\hat{\omega}(t)}{\partial p} > 0$ if and only if $\frac{\partial L(\hat{\omega}|t,p)}{\partial p}\big|_{\hat{\omega}=\hat{\omega}(t)} > 0$. Notice that

$$\frac{\partial L(\hat{\omega}|t,p)}{\partial p} = t_1^*\left(\frac{p-\alpha u(\hat{\omega},t)}{1-\alpha};\hat{\omega}\right)\left(\frac{1}{1-\alpha}\right) - t_1^*(p;\bar{\omega}(s)). \tag{A.23}$$

We first show that (A.23) is positive at the margin; i.e., for type $t = t^*(p;\bar{\omega}(s))$. In this case, $\hat{\omega}(t) = \bar{\omega}(s)$ and thus $u(\hat{\omega},t) = u(\bar{\omega}(s),t) = p$, implying that $t_1^*\left(\frac{p-\alpha u(\hat{\omega},t)}{1-\alpha};\hat{\omega}\right) = t_1^*(p;\bar{\omega}(s))$. Hence, (A.23) is positive if and only if $\alpha > 0$. To see why this condition must hold more generally, let $\hat{\omega}(t|p)$ denote the equilibrium perception of an agent with taste t facing price p, and consider p_0 and $p_1 > p_0$. Let $t_0^* \equiv t^*(p_0;\bar{\omega}(s))$. The preceding argument establishes that $\hat{\omega}(t_0^*|p_1) > \hat{\omega}(t_0^*|p_0)$. Furthermore, from Part 1, we know that $\hat{\omega}(t|p)$ is strictly decreasing in t for each $p \in \{p_0, p_1\}$. Since $\hat{\omega}(t_0^*|p_1) > \hat{\omega}(t_0^*|p_0)$, we must have $\hat{\omega}(t|p_1) > \hat{\omega}(t|p_0)$ for all t if $\hat{\omega}(\cdot|p_0)$ and $\hat{\omega}(\cdot|p_1)$ do not cross; that is, if there exists no $\tilde{t} \in \mathcal{T}$ such that $\hat{\omega}(\tilde{t}|p_1) = \hat{\omega}(\tilde{t}|p_0)$. Toward a contradiction, suppose such a $\tilde{t}$ exists, and let $\tilde{\omega} = \hat{\omega}(\tilde{t}|p_1) = \hat{\omega}(\tilde{t}|p_0)$. By definition, $\tilde{\omega}$ must rationalize the observed levels of demand at prices p_0 and p_1. But this contradicts the fact that the agent must infer distinct estimates of ω from these different levels of demand. Moreover, it is immediate that (A.23) is strictly positive, as desired, for the functional form for u in the main text since it implies that t_1^* is a constant. ■

Proof of Proposition 2. We prove this result for the more general class of utility functions introduced at the beginning of the proof of Proposition 1 (i.e.,$u(\omega,t)$ is strictly increasing and differentiable with respect to both variables, satisfies $\frac{\partial^2}{\partial\omega\partial t}u(\omega,t) \geq 0$, and is linear in ω). Thus, the results of the generalized version of Proposition 1 apply.

The random variable describing the valuations of the uninformed agents in the rational steady-state equilibrium is $v(T) \equiv u(\bar{\omega}(s),T)$. Under projection, this random variable is $\hat{v}(T) \equiv u(\hat{\omega}(T),T)$. We argue that $\hat{v}(\cdot)$ is a clockwise rotation of $v(\cdot)$. First, note that $\hat{v}(t^*) = u(\hat{\omega}(t^*),t^*) = u(\bar{\omega}(s),t^*) = v(t^*)$, which follows from the proof of Part 1 of Proposition 1 where we show that $\hat{\omega}(t^*) = \bar{\omega}(s)$. Thus, v and $\hat{v}$ intersect at t^*. Next, for $t > t^*$, $\hat{v}(t) = u(\hat{\omega}(t),t) < u(\bar{\omega}(s),t) = v(t)$ since $\hat{\omega}(t) < \bar{\omega}(s)$ for $t > t^*$ given that $\hat{\omega}(t)$ is strictly decreasing in t (as shown in Part 1 of Proposition 1). Similarly, for $t < t^*$, $\hat{v}(t) = u(\hat{\omega}(t),t) > u(\bar{\omega}(s),t) = v(t)$ since $\hat{\omega}(t) > \bar{\omega}(s)$ for $t < t^*$, which again follows from $\hat{\omega}(t)$ being strictly decreasing in t. Thus, $\hat{v}$ is a clockwise rotation of v. Since v and $\hat{v}$ are strictly increasing functions, this rotation property implies that $\hat{v}(T)$ is less disperse than $v(T)$ in the sense of the dispersion order defined by Shaked and Shanthikumar (2007) (see the end of the proof of Proposition B.1 in Appendix B.1 for the definition of this order). Thus, by Theorem 3.B.16 of Shaked and Shanthikumar (2007), $\mathrm{Var}\big(\hat{v}(T)\big) < \mathrm{Var}\big(v(T)\big)$. ■

Proof of Proposition 3. To show that $D(p_2; \bar{\omega}_2, \bar{\omega}(s))$ is a counter-clockwise rotation of $D^I(p_2; \bar{\omega}(s))$ about the point (p_1, d_1), we must show that: (i) $D(p_1; \bar{\omega}_2, \bar{\omega}(s)) = D^I(p_1; \bar{\omega}(s))$; (ii) $D(p_2; \bar{\omega}_2, \bar{\omega}(s)) > D^I(p_2; \bar{\omega}(s))$ for all $p_2 < p_1$; and (iii) $D(p_2; \bar{\omega}_2, \bar{\omega}(s)) < D^I(p_2; \bar{\omega}(s))$ for all $p_2 > p_1$.

First, from (12), $D(p_1; \bar{\omega}_2, \bar{\omega}(s)) = D^I(p_1; \bar{\omega}(s)) \Leftrightarrow D^U(p_1; \bar{\omega}_2) = D^I(p_1; \bar{\omega}(s))$. Since $\bar{\omega}_2 = (1-\alpha)\bar{\omega}(s) + \alpha p_1$, substituting this value into (11) yields $D^U(p_1; \bar{\omega}_2) = 1 - F(p_1 - \bar{\omega}(s))$, which is identical to $D^I(p_1; \bar{\omega}(s))$. Next, note that $D(p_2; \bar{\omega}_2, \bar{\omega}(s)) > D^I(p_2; \bar{\omega}(s)) \Leftrightarrow D^U(p_2; \bar{\omega}_2) > D^I(p_2; \bar{\omega}(s))$, which is equivalent to $F\left(\frac{p_2 - \bar{\omega}_2}{1-\alpha}\right) < F(p_2 - \bar{\omega}(s))$. Using the definition of $\bar{\omega}_2$, the preceding inequality can be written as

$$F\left(p_2 - \bar{\omega}(s) + \frac{\alpha(p_2 - p_1)}{1-\alpha}\right) < F(p_2 - \bar{\omega}(s)). \tag{A.24}$$

Thus, (A.24) implies that $D^U(p_2; \bar{\omega}_2) > D^I(p_2; \bar{\omega}(s))$ if $p_2 < p_1$. It additionally implies that if $p_2 > p_1$, then $D^U(p_2; \bar{\omega}_2) < D^I(p_2; \bar{\omega}(s))$ and thus $D(p_2; \bar{\omega}_2, \bar{\omega}(s)) < D^I(p_2; \bar{\omega}(s))$. ■

Proof of Proposition 4. *Part 1.* The seller's objective is

$$\max_{p_1, p_2} \Pi(p_1, p_2; \alpha, \lambda), \tag{A.25}$$

subject to the dynamic constraint $\bar{\omega}_2 = \alpha p_1 + (1-\alpha)\bar{\omega}(s)$. Note that

$$\Pi(p_1, p_2; \alpha, \lambda) = p_1 D_1(p_1; \bar{\omega}(s)) + p_2 D_2(p_2; \bar{\omega}_2, \bar{\omega}(s)), \tag{A.26}$$

where, from Equation (12), we have

$$\begin{aligned} D_1(p_1; \bar{\omega}(s)) &= D^I(p; \bar{\omega}(s)), & \text{(A.27)} \\ D_2(p_2; \bar{\omega}_2, \bar{\omega}(s)) &= \lambda D^I(p_2; \bar{\omega}(s)) + (1-\lambda) D^U(p_2; \bar{\omega}_2), & \text{(A.28)} \end{aligned}$$

with $D^I(p; \bar{\omega}(s)) = 1 - F(p - \bar{\omega}(s))$ and $D^U(p; \bar{\omega}_2) = 1 - F\left(\frac{p - \bar{\omega}_2}{1-\alpha}\right)$.

Potential Cases and Outline. We first describe the potential mix of interior and corner solutions and argue which of these are possible at the optimum. Then, for each possible case, we proceed to show that $p_1^* > p^M$ and $p_1^* > p_2^*$.

Fixing s, let $\underline{v} = \bar{\omega}(s) + \underline{t}$ and $\overline{v} = \bar{\omega}(s) + \bar{t}$ denote the expected valuations of the lowest and highest informed types, respective. The set of valuations among informed types is thus $\mathcal{V} = [\underline{v}, \overline{v}]$. As a function of p_1, an uninformed consumer's valuation in period 2 is $(1-\alpha)(\bar{\omega}(s) + t) + \alpha p_1$. Notice that at any optimum, $p_1 \in [\underline{v}, \bar{p}]$, where, recall, the price ceiling is $\bar{p} = \overline{v} - \kappa$ for some $\kappa > 0$ arbitrarily small such that $\bar{p} > p^M$. Hence, given p_1 and $\alpha > 0$, the set of valuations of uninformed consumers in period 2, denoted $\widehat{\mathcal{V}} \equiv [(1-\alpha)\underline{v} + \alpha p_1, (1-\alpha)\overline{v} + \alpha p_1]$, is a strict subset of $\mathcal{V}$.

First, notice that it is never optimal for the seller to serve all consumers in period 1. Since (p^M, s) admits interior demand, it is not optimal to serve all consumers in the rational benchmark; moreover, doing so under projection leads to the least attractive distribution of perceived valuations in period 2. Hence, in period 1 we either have an interior solution or a price equal to the price ceiling: $p_1^* \in (\underline{v}, \bar{p}]$.

Now consider possible corner cases in period 2. Since the valuations of uninformed types are a strict subset of the valuations of informed types, demand in period 2 is $D_2(p;\bar{\omega}_2;\bar{\omega}(s)) =$

$$\begin{cases} \lambda D^I(p;\bar{\omega}(s)) + (1-\lambda) & \text{if} \quad p \in [\underline{v}, (1-\alpha)\underline{v} + \alpha p_1), \\ \lambda D^I(p;\bar{\omega}(s)) + (1-\lambda) D^U(p;\bar{\omega}_2) & \text{if} \quad p \in [(1-\alpha)\underline{v} + \alpha p_1, (1-\alpha)\overline{v} + \alpha p_1], \\ \lambda D^I(p;\bar{\omega}(s)) & \text{if} \quad p \in ((1-\alpha)\overline{v} + \alpha p_1, \bar{p}]. \end{cases} \tag{A.29}$$

We now argue that the seller will never operate strictly within the first or third region of the demand function above, but may operate at the corner $p_2^c \equiv (1-\alpha)\underline{v} + \alpha p_1$ at which all uninformed types are served. First consider the third region. It is clearly sub-optimal to serve only informed types in period 2 since the strategy $p_1 = p_2 = p^M$ yields the seller the rational static monopoly profit in each period. Thus, deviating from these prices would require the seller to strictly benefit by serving consumers with manipulated beliefs, which is not possible when serving only informed types. Now consider the interior of the first region, where the seller sets a price below the lowest perceived valuation of uninformed types. This cannot happen at the optimum since it involves using p_1 to inflate the beliefs of uninformed types to an inefficient extent: since all uninformed types strictly prefer to buy at p_2 given $\bar{\omega}_2$, a slight reduction in p_1 would have no effect on the demand of the uninformed (or informed) agents in period 2 but would strictly increase the seller's profit in period 1. Thus, $p_2^* \geq p_2^c$ and in period 2 we either have an interior solution (in the middle region of A.29) or the corner solution such that $p_2^* = p_2^c$.

Next, we show that $p_1^* > p^M$ and $p_1^* > p_2^*$ in any of the possible cases noted above (i.e., interior or ceiling in period 1, and interior or corner in period 2).

Case 1: Interior Solutions. Substituting the dynamic constraint into D_2 in (A.28), the first-order conditions of (A.25) are:

$$\frac{\partial}{\partial p_1} p_1 D_1(p_1;\bar{\omega}(s)) + p_2 \frac{\partial}{\partial \bar{\omega}_2} D_2(p_2;\bar{\omega}_2,\bar{\omega}(s)) \frac{\partial \bar{\omega}_2}{\partial p_1} = 0 \quad \text{and} \quad \frac{\partial}{\partial p_2} p_2 D_2(p_2;\bar{\omega}_2,\bar{\omega}(s)) = 0. \tag{A.30}$$

Define the following functions, each corresponding to the price derivative of the seller's profit in period n with respect to p_n for $n = 1, 2$:

$$M_1(p;\bar{\omega}(s)) \equiv \frac{\partial}{\partial p} p D_1(p;\bar{\omega}(s)) \quad \text{and} \quad M_2(p;\bar{\omega}_2,\bar{\omega}(s)) \equiv \frac{\partial}{\partial p} p D_2(p;\bar{\omega}_2,\bar{\omega}(s)). \tag{A.31}$$

Substituting these expressions along with the relevant derivatives into the FOCs in (A.30) yields:

$$M_1(p_1;\bar{\omega}(s)) + p_2 \left(\frac{\alpha(1-\lambda)}{1-\alpha}\right) f\left(\frac{p_2 - \bar{\omega}_2}{1-\alpha}\right) = 0, \tag{A.32}$$

$$M_2(p_2;\bar{\omega}_2,\bar{\omega}(s)) = 0. \tag{A.33}$$

Step 1: $p_1^* > p^M$. Since (p^M, s) admits interior demand under rational inference and since f being log-concave implies that F has an increasing hazard rate, M_1 is strictly decreasing in p and has exactly one root at $p^M > 0$. Note that FOC (A.32) implies that p_1^* solves

$$M_1(p_1^*;\bar{\omega}(s)) = -p_2^* \left(\frac{\alpha(1-\lambda)}{1-\alpha}\right) f\left(\frac{p_2^* - \bar{\omega}_2}{1-\alpha}\right), \tag{A.34}$$

where the right-hand side is strictly negative at an interior solution whenever $\alpha > 0$. Thus, since M_1 is decreasing in p and $M_1(p^M; \bar{\omega}(s)) = 0$, we must have $p_1^* > p^M$.

Step 2: $p_2^* < p_1^*$. FOC (A.33) implies that p_2^* solves $M_2(p_2^*; \bar{\omega}_2, \bar{\omega}(s)) = 0$. Toward a contradiction, suppose that $p_2^* = p_1^*$. We argue that $M_2(p_1^*; \bar{\omega}_2, \bar{\omega}(s)) < M_1(p_1^*; \bar{\omega}(s))$. Note that

$$M_2(p; \bar{\omega}_2, \bar{\omega}(s)) = D_2(p; \bar{\omega}_2, \bar{\omega}(s)) - p\left[\lambda f(p - \bar{\omega}(s)) + \left(\frac{1-\lambda}{1-\alpha}\right) f\left(\frac{p - \bar{\omega}_2}{1-\alpha}\right)\right]. \tag{A.35}$$

At $p = p_1^*$, we have $\bar{\omega}_2 = (1-\alpha)\bar{\omega}(s) + \alpha p_1^*$ and $(p - \bar{\omega}_2)/(1-\alpha) = p_1^* - \bar{\omega}(s)$, which further implies that $D_2(p_1^*; \bar{\omega}_2, \bar{\omega}(s)) = 1 - F(p_1^* - \bar{\omega}(s)) = D_1(p_1^*; \bar{\omega}(s))$. Thus, evaluating M_2 at $p = p_1^*$ yields $M_2(p_1^*; \bar{\omega}_2, \bar{\omega}(s)) = D_1(p_1^*; \bar{\omega}(s)) - p_1^* f(p_1^* - \bar{\omega}(s)) \left(\frac{1-\alpha\lambda}{1-\alpha}\right)$. However, note that $M_1(p_1^*; \bar{\omega}(s)) = D_1(p_1^*; \bar{\omega}(s)) - p_1^* f(p_1^* - \bar{\omega}(s))$, and thus $M_2(p_1^*; \bar{\omega}_2, \bar{\omega}(s)) < M_1(p_1^*; \bar{\omega}(s))$ if and only if $-p_1^* f(p_1^* - \bar{\omega}(s)) \left(\frac{1-\alpha\lambda}{1-\alpha}\right) < -p_1^* f(p_1^* - \bar{\omega}(s))$, which holds for any $\alpha > 0$. However, this presents a contradiction: since $M_1(p_1^*; \bar{\omega}(s)) < 0$ by FOC (A.32), $M_2(p_1^*; \bar{\omega}_2, \bar{\omega}(s)) < M_1(p_1^*; \bar{\omega}(s)) \Rightarrow M_2(p_1^*; \bar{\omega}_2, \bar{\omega}(s)) < 0$, which violates FOC (A.33). Thus, if $M_2(p; \bar{\omega}_2, \bar{\omega}(s))$ is decreasing in p, we must have $p_2^* < p_1^*$ in order for both FOCs to hold. To complete the proof, we only need to show that $M_2(p; \bar{\omega}_2, \bar{\omega}(s))$ is decreasing in p.

Step 3: M_2 is decreasing in p. Notice that

$$\begin{aligned} M_2(p; \bar{\omega}_2, \bar{\omega}(s)) &= \lambda \underbrace{\left[\frac{\partial}{\partial p} p D^I(p; \bar{\omega}(s))\right]}_{\equiv M^I(p; \bar{\omega}(s))} + (1-\lambda) \underbrace{\left[\frac{\partial}{\partial p} p D^U(p; \bar{\omega}_2)\right]}_{\equiv M^U(p; \bar{\omega}_2)} \\ &= \lambda M^I(p; \bar{\omega}(s)) + (1-\lambda) M^U(p; \bar{\omega}_2). \end{aligned} \tag{A.36}$$

It is immediate that $M^I(p; \bar{\omega}(s)) = M_1(p; \bar{\omega}(s))$ and is hence decreasing in p. We can also show that M^U is decreasing in p given our assumptions on F. The following Lemma establishes this.

Lemma A.2. *Suppose the family of distributions $\{F(x - \bar{\omega})\}_{\bar{\omega} \in \mathbb{R}}$ is such that for any $\bar{\omega}(s)$, $M^I(p; \bar{\omega}(s)) \equiv \frac{\partial}{\partial p} p[1 - F(p - \bar{\omega}(s))]$ is decreasing at all p such that $F(p - \bar{\omega}(s)) \in (0,1)$. Then for any $\alpha \in [0,1)$ and $\bar{\omega}_2 \in \mathbb{R}$, $M^U(p; \bar{\omega}_2) \equiv \frac{\partial}{\partial p} p[1 - F(\frac{p - \bar{\omega}_2}{1-\alpha})]$ is decreasing at all p such that $F(\frac{p - \bar{\omega}_2}{1-\alpha}) \in (0,1)$.*

We now prove Lemma A.2. Consider an arbitrary value of $\bar{\omega}(s) \in \mathbb{R}$. Notice that $M^I(p; \bar{\omega}(s)) = 1 - F(p - \bar{\omega}(s)) - pf(p - \bar{\omega}(s))$, and hence the assumption of the lemma implies $\frac{\partial}{\partial p} M^I(p; \bar{\omega}(s)) < 0 \Leftrightarrow -f(p - \bar{\omega}(s)) - f(p - \bar{\omega}(s)) - pf'(p - \bar{\omega}(s))$ on the relevant domain, which is equivalent to

$$-2f(p - \bar{\omega}(s)) - pf'(p - \bar{\omega}(s)) \leq 0 \tag{A.37}$$

for all $\bar{\omega}(s)$ (and strictly so for $p - \bar{\omega}(s)$ on the interior of the support of F). Now note that $M^U(p; \bar{\omega}_2) \equiv \frac{\partial}{\partial p} p[1 - F(\frac{p - \bar{\omega}_2}{1-\alpha})] = 1 - F(\frac{p - \bar{\omega}_2}{1-\alpha}) - pf(\frac{p - \bar{\omega}_2}{1-\alpha}) \frac{1}{1-\alpha}$. To show that $M^U(p; \bar{\omega}(s))$ is decreasing in p, note that $\frac{\partial}{\partial p} M^U(p; \bar{\omega}_2) = -2f\left(\frac{p - \bar{\omega}_2}{1-\alpha}\right) \frac{1}{1-\alpha} - pf'\left(\frac{p - \bar{\omega}_2}{1-\alpha}\right) \frac{1}{(1-\alpha)^2}$. The previous expression is weakly negative if and only if

$$-2f\left(\frac{p - \bar{\omega}_2}{1-\alpha}\right) - pf'\left(\frac{p - \bar{\omega}_2}{1-\alpha}\right) \frac{1}{(1-\alpha)} \leq 0. \tag{A.38}$$

Under a change of variables with $\tilde{p} = \frac{p}{1-\alpha}$ and $\tilde{\omega} = \frac{\bar{\omega}_2}{1-\alpha}$, the previous condition is equivalent to

$$-2f(\tilde{p} - \tilde{\omega}) - \tilde{p}f'(\tilde{p} - \tilde{\omega}) \leq 0. \tag{A.39}$$

This condition is equivalent to Condition (A.37), which holds by assumption. Furthermore, Condition (A.37) additionally implies that Condition (A.39) holds with a strict inequality when $\frac{p-\bar{\omega}_2}{1-\alpha}$ is on the interior of the support of F. This completes the proof of Lemma A.2.

Since F satisfies the assumption of Lemma A.2 (because log-concavity of f implies that F has an increasing hazard rate), M^U is decreasing and thus M_2 is decreasing since it is the convex combination of decreasing functions. This completes Case 1.

Case 2: $p_1^* = \bar{p}$. Suppose the optimal price in period 1 is the price ceiling. Then $p_1^* > p^M$ given that $\bar{p} > p^M$. To show $p_1^* > p_2^*$, suppose that $p_2^* = \bar{p}$ for a contradiction. Recall that if $p_1 = p_2$, then $D^U(p_2; \bar{\omega}_2) = D^I(p_2; \bar{\omega}(s)) \Rightarrow D_2(p_2; \bar{\omega}_2, \bar{\omega}(s)) = D^I(p_2; \bar{\omega}(s))$. Thus, the seller's total profit from $p_1^* = p_2^* = \bar{p}$ would be $2D^I(\bar{p}; \bar{\omega}(s)) < 2D^I(p^M; \bar{\omega}(s))$ since p^M uniquely maximizes $pD^I(p; \bar{\omega}(s))$. Thus, $p_1 = p_2 = p^M$ is strictly preferred to $p_1^* = p_2^* = \bar{p}$, contradicting the presumption that the latter path is optimal. Thus, we must have $p_2^* < p_1^*$.

Case 3: p_1^* *interior yet* $p_2^* = p_2^c$. In this case, $p_2^* = p_2^c = (1-\alpha)\underline{v} + \alpha p_1^*$. Note that $p_1^* > p_2^* \Leftrightarrow p_1^* > \underline{v}$, which is true given that is sub-optimal to serve all consumers in period 1. Thus, we need to show that $p_1^* > p^M$ when p_1^* is interior (the ceiling case is considered above). The seller chooses p_1^* to maximize $p_1 D^I(p_1; \bar{\omega}(s)) + p_2^c[\lambda D^I(p_2; \bar{\omega}(s)) + 1 - \lambda]$, yielding a FOC of

$$M_1(p_1^*; \bar{\omega}(s)) + \alpha[\lambda D^I(p_2^c; \bar{\omega}(s)) + 1 - \lambda - \lambda p_2^c f(p_2^c - \bar{\omega}(s))] = 0, \tag{A.40}$$

and thus

$$M_1(p_1^*; \bar{\omega}(s)) + \alpha\lambda M_1(p_2^c; \bar{\omega}(s)) + \alpha(1-\lambda) = 0. \tag{A.41}$$

Recall that $M_1(p^M; \bar{\omega}(s)) = 0$ and $M_1(p; \bar{\omega}(s)) > 0$ for all $p < p^M$. Thus, since $p_2^c < p_1^*$, if $p_1^* \leq p^M$, then $M_1(p_1^*; \bar{\omega}(s)) + \alpha\lambda M_1(p_2^c; \bar{\omega}(s)) > 0$, contradicting the FOC above. This completes the proof of Part 1.

Part 2. Effect of α. First consider the case in which p_1^* and p_2^* are interior solutions to the optimization program in (A.26). From the Envelope Theorem, $\frac{\partial p_n^*}{\partial \alpha} = 0$ for $n = 1, 2$, and hence

$$\frac{\partial}{\partial \alpha}\Pi(p_1, p_2; \alpha, \lambda) = -p_2^* \left[\lambda f(t_2^*)\frac{\partial t_2^*}{\partial \alpha} + (1-\lambda) f\left(\hat{t}_2\right) \frac{\partial \hat{t}_2}{\partial \alpha}\right], \tag{A.42}$$

where we've defined $t_2^* \equiv p_2 - \bar{\omega}(s)$ and $\hat{t}_2 \equiv \frac{p_2^* - (1-\alpha)\bar{\omega}(s) - \alpha p_1^*}{1-\alpha}$. Since t_2^* is the marginal informed type, $\frac{\partial t_2^*}{\partial \alpha} = 0$. Now note that

$$\frac{\partial \hat{t}_2}{\partial \alpha} = \frac{(1-\alpha)[\bar{\omega}(s) - p_1^*] + [p_2^* - (1-\alpha)\bar{\omega}(s) - \alpha p_1^*]}{(1-\alpha)^2} = -\frac{p_1^* - p_2^*}{(1-\alpha)^2}. \tag{A.43}$$

Substituting these values back into (A.42) yields

$$\frac{\partial}{\partial \alpha}\Pi(p_1, p_2; \alpha, \lambda) = (1-\lambda)p_2^* f\left(\frac{p_2^* - (1-\alpha)\bar{\omega}_1 - \alpha p_1^*}{1-\alpha}\right)\left[\frac{p_1^* - p_2^*}{(1-\alpha)^2}\right]. \tag{A.44}$$

Since $\lambda < 1$, the expression above is positive whenever $p_1^* > p_2^*$, which is true by Part 1 of this proposition. The case in which $p_1^* = \bar{p}$ and p_2^* is interior yields for $\frac{\partial}{\partial \alpha}\Pi(p_1, p_2; \alpha, \lambda)$ an expression that is identical to expression (A.44). Finally, consider the case in which $p_2^* = p_2^c = (1-\alpha)\underline{v} + \alpha p_1$ (i.e., the corner case described in Part 1 in which all uninformed types are served in period 2). In period 1, the seller chooses p_1 to maximize

$$\Pi^c(p_1; \alpha, \lambda) = p_1[1 - F(p_1 - \bar{\omega}(s))] + [(1-\alpha)\underline{v} + \alpha p_1]\Bigg[1 - \lambda F((1-\alpha)\underline{v} + \alpha p_1 - \bar{\omega}(s))\Bigg]. \quad \text{(A.45)}$$

Note that this profit function accounts for the fact that all uninformed types buy in period 2. Let p_1^* be the value of p_1 that maximizes the expression above, and let $p_2^c(p_1^*) \equiv (1-\alpha)\underline{v} + \alpha p_1^*$. For either an interior value of p_1^* or $p_1^* = \bar{p}$, we have

$$\frac{\partial \Pi^c(p_1; \alpha, \lambda)}{\partial \alpha} = (p_1^* - \underline{v})\Bigg[1 - \lambda F(p_2^c(p_1^*) - \bar{\omega}(s))\Bigg] - p_2^c(p_1^*)\lambda f\Bigg(p_2^c(p_1) - \bar{\omega}(s)\Bigg)(p_1^* - \underline{v}); \quad \text{(A.46)}$$

thus, $\frac{\partial}{\partial \alpha}\Pi^c(p_1; \alpha, \lambda) > 0$ if and only if $[1 - \lambda F(p_2^c(p_1^*) - \bar{\omega}(s))] - p_2^c(p_1^*)\lambda f\,(p_2^c(p_1^*) - \bar{\omega}(s)) > 0$. The previous condition must hold given that we are focusing on the case in which all uninformed types are served: as argued above, it is optimal to set the highest possible price in the first region of D_2 in (A.29), and hence the previous inequality must hold for all $p_2 \leq (1-\alpha)\underline{v} + \alpha p_1$.

Effect of λ. Similar to the approach above, if p_2^* is interior and either p_1^* is interior or $p_1^* = \bar{p}$, then we have

$$\frac{\partial}{\partial \lambda}\Pi(p_1, p_2; \alpha, \lambda) = p_2^*\left[-F(t_2^*) + F\left(\hat{t}_2\right)\right], \quad \text{(A.47)}$$

where neither t_2^* nor $\hat{t}_2$ depend on λ. This expression is negative whenever $\hat{t}_2 < t_2^*$. Notice that

$$\hat{t}_2 = \frac{p_2^* - (1-\alpha)\bar{\omega}(s) - \alpha p_1^*}{1-\alpha} = p_2^* - \bar{\omega}(s) - \frac{\alpha}{1-\alpha}[p_1^* - p_2^*] = t_2^* - \frac{\alpha}{1-\alpha}[p_1^* - p_2^*]. \quad \text{(A.48)}$$

Since $\alpha > 0$, $\hat{t}_2 < t_2^* \Leftrightarrow p_1^* - p_2^* > 0$, which is again true by Part 1 of this proposition. If instead we have a corner solution in period 2, then the profit function is as in (A.45) and

$$\frac{\partial \Pi^c(p_1; \alpha, \lambda)}{\partial \lambda} = -p_2^c(p_1^*)F\Bigg(p_2^c(p_1) - \bar{\omega}(s)\Bigg), \quad \text{(A.49)}$$

which is clearly negative. ■

Proof of Proposition 5. *Part 1.* Consider the optimal price pair (p_1^*, p_2^*). Let $t_2^* \equiv p_2^* - \bar{\omega}(s)$ denote the marginal informed type in period 2, and and let $\hat{t}_2 \equiv \frac{p_2^* - \bar{\omega}_2}{1-\alpha}$ denote the marginal uninformed type. Note that if $\hat{t}_2 < t_2^*$, then the interval of types who adopt the good in period 2 at a price above their true expected valuation is $[\hat{t}_2, t_2^*]$. From (A.48), we have $t_2^* - \hat{t}_2 = \frac{\alpha}{1-\alpha}[p_1^* - p_2^*]$. Since $p_1^* - p_2^* > 0$ for all $\alpha > 0$ (by Proposition 4 Part 1), we know that $\hat{t}_2 < t_2^*$. Thus, the width of the interval of types who wrongly adopt is $t_2^* - \hat{t}_2 = \frac{\alpha}{1-\alpha}[p_1^* - p_2^*]$, which is strictly positive.

Part 2. Suppose that $\bar{\omega}(s) + \underline{t} < 0$. We show that an α sufficiently large will induce the seller to set the "corner" price in period 2 at which all uninformed types are served. Recall from the proof of Proposition 4 that this price is $p_2^c = (1-\alpha)\underline{v} + \alpha p_1$, where $\underline{v} = \bar{\omega}(s) + \underline{t}$. We will show that the price derivative of the period-2 profit function is necessarily negative at p_2^c for α sufficiently large, implying that $p_2^* = p_2^c$, and thus that all uninformed types are served. To this end, denote the period-2 profit by

$$\Pi_2(p_2;p_1) = p_2\Bigg(1-\lambda F(p_2-\bar{\omega}(s)) - (1-\lambda)F\left(\frac{p_2-(1-\alpha)\bar{\omega}(s)-\alpha p_1}{1-\alpha}\right)\Bigg). \tag{A.50}$$

Hence, we have

$$\begin{aligned}\frac{\partial \Pi_2(p_2;p_1)}{\partial p_2} = &\left(1-\lambda F(p_2-\bar{\omega}(s)) - (1-\lambda)F\left(\frac{p_2-(1-\alpha)\bar{\omega}(s)-\alpha p_1}{1-\alpha}\right)\right)\\ &- p_2\Bigg(\lambda f(p_2-\bar{\omega}(s)) + \frac{(1-\lambda)}{(1-\alpha)}f\left(\frac{p_2-(1-\alpha)\bar{\omega}(s)-\alpha p_1}{1-\alpha}\right)\Bigg).\end{aligned} \tag{A.51}$$

To evaluate $\frac{\partial \Pi_2(p_2;p_1)}{\partial p_2}\Big|_{p_2=p_2^c}$, notice that $\frac{p_2^c-(1-\alpha)\bar{\omega}(s)-\alpha p_1}{1-\alpha} = \underline{t}$. Since $F(\underline{t})=0$, we have

$$\frac{\partial \Pi_2(p_2;p_1)}{\partial p_2}\Bigg|_{p_2=p_2^c} = 1-\lambda\Big(F(p_2^c-\bar{\omega}(s)) - p_2^c f(p_2^c-\bar{\omega}(s))\Big) - p_c^2\frac{(1-\lambda)}{(1-\alpha)}f\left(\underline{t}\right), \tag{A.52}$$

and thus a sufficient condition for $\frac{\partial \Pi_2(p_2;p_1)}{\partial p_2}\Big|_{p_2=p_2^c} < 0$ is $p_c^2\frac{(1-\lambda)}{(1-\alpha)}f\left(\underline{t}\right) > 1$. Since $p_2^c = (1-\alpha)\underline{v} + \alpha p_1$, the previous sufficient condition is equivalent to

$$\underline{v} + \frac{\alpha}{(1-\alpha)}p_1 > \frac{1}{(1-\lambda)f(\underline{t})}. \tag{A.53}$$

From Proposition 4 Part 1, we know that along the optimal price path, $p_1 > p^M$ for all $\alpha > 0$. Hence, a sufficient condition for (A.53) is

$$\underline{v} + \frac{\alpha}{(1-\alpha)}p^M > \frac{1}{(1-\lambda)f(\underline{t})}. \tag{A.54}$$

The right-hand side of (A.54) is positive and finite given that f is positive on $\mathcal{T}$. Thus, since $p^M > 0$, there exists $\tilde{\alpha} \in (0,1)$ such that $\underline{v} + \frac{\tilde{\alpha}}{(1-\tilde{\alpha})}p^M = \frac{1}{(1-\lambda)f(\underline{t})}$. Then $\alpha > \tilde{\alpha}$ implies that Condition (A.54) holds, and hence the seller chooses p_2^c such that all uninformed types are served in period 2. ■

Proof of Proposition 6. *Part 1.* We prove the claim by induction on $n = 2,\ldots,N$. As argued in the main text preceding Equation (11), $\hat{\omega}_2(t) = \bar{\omega}_2 - \alpha t$ for some $\bar{\omega}_2$ independent of t. This establishes the base case. Now suppose that in period n, $\hat{\omega}_n(t) = \bar{\omega}_n - \alpha t$. The marginal uninformed type in period n has taste $\hat{t}_n$ such that $\hat{\omega}_n(\hat{t}_n) + \hat{t}_n = p_n \Rightarrow \hat{t}_n = (p_n - \bar{\omega}_n)/(1-\alpha)$ and thus aggregate demand in period n is $d_n = \lambda\big[1-F(p_n-\bar{\omega}(s))\big] + (1-\lambda)\left[1-F\left(\frac{p_n-\bar{\omega}_n}{1-\alpha}\right)\right]$. An observer in generation $n+1$ then forms a perception of ω equal to $\hat{\omega}_{n+1}(t)$ such that $d_n = 1-\widehat{F}(p_n-\hat{\omega}_{n+1}(t)) = 1-F\left(\frac{p_n-\hat{\omega}_{n+1}(t)-\alpha t}{1-\alpha}\right) \Rightarrow \hat{\omega}_{n+1}(t) = [p_n-(1-\alpha)F^{-1}(1-d_n)] - \alpha t = \bar{\omega}_{n+1} - \alpha t$, where $\bar{\omega}_{n+1} \equiv p_n - (1-\alpha)F^{-1}(1-d_n)$ is independent of t.

Part 2. From the previous part, $\hat{\omega}_n(t) = \bar{\omega}_n - \alpha t$, and thus the quantity demanded in each period n, d_n, can be written in terms of $\bar{\omega}_n$ analogously to expression (12):

$$d_n = D(p_n;\bar{\omega}_n,\bar{\omega}(s)) \equiv \lambda\big[1-F(p_n-\bar{\omega}(s))\big] + (1-\lambda)\left[1-F\left(\frac{p_n-\bar{\omega}_n}{1-\alpha}\right)\right]. \tag{A.55}$$

An uninformed consumer in period $n+1$ with taste t thinks d_n is determined by $\widehat{D}^I(p_n;\hat{\omega}_{n+1}(t)|t)$ as in (9). Substituting $\hat{\omega}_{n+1}(t) = \bar{\omega}_{n+1} - \alpha t$ into $\widehat{D}^I(p_n;\hat{\omega}_{n+1}(t)|t)$ implies that an uninformed consumer in period $n+1$ thinks that d_n is determined by the following function of $\bar{\omega}_n$: $\widehat{D}(p_n;\bar{\omega}_{n+1}) \equiv 1 - F\left(\frac{p_n - \bar{\omega}_{n+1}}{1-\alpha}\right)$. Since the beliefs formed in $n+1$ must be consistent with d_n for all $n \geq 2$, we must have $d_n = \widehat{D}(p_n;\bar{\omega}_{n+1})$. Hence, the law of motion describing the process $(\bar{\omega}_n)$ is characterized by the condition

$$\widehat{D}(p_n;\bar{\omega}_{n+1}) = D(p_n;\bar{\omega}_n, \bar{\omega}(s)), \tag{A.56}$$

starting from the initial condition of $\bar{\omega}_2 = (1-\alpha)\bar{\omega}(s) + \alpha p_1$.

We are now ready to prove the second part of the proposition. The statement is readily verified for $n = 2$. For $n \geq 3$, the proof proceeds in two steps. First, we will show that $\bar{\omega}_n$ is increasing in p_{n-1}. Second, we will show that $\overline{\omega}_n$ is increasing in $\bar{\omega}_{n-1}$. Combining these two results then implies that $\bar{\omega}_n$ is increasing in p_k for all $n \geq 3$ and $k < n$.

Step 1: *$\bar{\omega}_n$ is increasing in p_{n-1}*. From Equation (A.56), $\bar{\omega}_n$ is the unique value that solves $\widehat{D}\left(p_{n-1};\bar{\omega}_n\right) = D\left(p_{n-1};\bar{\omega}_{n-1},\bar{\omega}\left(s\right)\right)$, which is equivalent to

$$1 - F\left(\frac{p_{n-1} - \bar{\omega}_n}{1-\alpha}\right) = \lambda\left[1 - F\left(p_{n-1} - \bar{\omega}\left(s\right)\right)\right] + (1-\lambda)\left[1 - F\left(\frac{p_{n-1} - \bar{\omega}_{n-1}}{1-\alpha}\right)\right].$$

Solving this expression for $\bar{\omega}_n$ yields

$$\bar{\omega}_n = p_{n-1} - (1-\alpha)\,F^{-1}\left(\lambda F\left(p_{n-1} - \bar{\omega}\left(s\right)\right) + (1-\lambda)\,F\left(\frac{p_{n-1} - \bar{\omega}_{n-1}}{1-\alpha}\right)\right). \tag{A.57}$$

Differentiating (A.57) with respect to p_{n-1} then yields

$$\frac{\partial \bar{\omega}_n}{\partial p_{n-1}} = 1 - (1-\alpha)\left[\frac{\lambda f\left(p_{n-1} - \bar{\omega}\left(s\right)\right) + \left(\frac{1-\lambda}{1-\alpha}\right) f\left(\frac{p_{n-1} - \bar{\omega}_{n-1}}{1-\alpha}\right)}{f\left(F^{-1}\left(\lambda F\left(p_{n-1} - \bar{\omega}\left(s\right)\right) + (1-\lambda)\,F\left(\frac{p_{n-1} - \bar{\omega}_{n-1}}{1-\alpha}\right)\right)\right)}\right]. \tag{A.58}$$

Let $t^*_{n-1} := p_{n-1} - \bar{\omega}\left(s\right)$ and $\hat{t}_{n-1} := \frac{p_{n-1} - \bar{\omega}_{n-1}}{1-\alpha}$; these denote the type of the marginal informed and uninformed buyer in period $n-1$, respectively. Also let

$$\phi_{n-1}\left(\lambda\right) \equiv f\left(F^{-1}\left(\lambda F\left(t^*_{n-1}\right) + (1-\lambda)\,F\left(\hat{t}_{n-1}\right)\right)\right) - (1-\alpha)\,\lambda f\left(t^*_{n-1}\right) - (1-\lambda)\,f\left(\hat{t}_{n-1}\right).$$

Then Equation (A.58) implies that $\frac{\partial \bar{\omega}_n}{\partial p_{n-1}} > 0 \Leftrightarrow\ \phi_{n-1}\left(\lambda\right) > 0$.

Next, we show that the function $\phi_{n-1}\left(\lambda\right)$ is strictly positive for any $\lambda \in (0,1]$. Begin by noticing that $\phi_{n-1}\left(0\right) = 0 < \alpha f\left(t^*_{n-1}\right) = \phi_{n-1}\left(1\right)$. Moreover, we have

$$\begin{aligned}\phi'_{n-1}\left(\lambda\right) = \frac{f'\left(F^{-1}\left(\lambda F\left(t^*_{n-1}\right) + (1-\lambda)\,F\left(\hat{t}_{n-1}\right)\right)\right)}{f\left(F^{-1}\left(\lambda F\left(t^*_{n-1}\right) + (1-\lambda)\,F\left(\hat{t}_{n-1}\right)\right)\right)}&\left[F\left(t^*_{n-1}\right) - F\left(\hat{t}_{n-1}\right)\right] \\ &- (1-\alpha)\,f\left(t^*_{n-1}\right) + f\left(\hat{t}_{n-1}\right).\end{aligned} \tag{A.59}$$

Notice that neither the sign of $-\left(1-\alpha\right)f\left(t^*_{n-1}\right) + f\left(\hat{t}_{n-1}\right)$ nor that of $F\left(t^*_{n-1}\right) - F\left(\hat{t}_{n-1}\right)$ depend on λ. Moreover, since f is log-concave, the ratio $\frac{f'}{f}$ is decreasing. Hence, $\phi'_{n-1}\left(\lambda\right)$ can change sign at most once. Additionally, we have

$$\phi'_{n-1}(\lambda)\big|_{\lambda=0} = \frac{f'\left(\hat{t}_{n-1}\right)}{f\left(\hat{t}_{n-1}\right)}\left[F\left(t^*_{n-1}\right) - F\left(\hat{t}_{n-1}\right)\right] - (1-\alpha) f\left(t^*_{n-1}\right) + f\left(\hat{t}_{n-1}\right)$$

and

$$\phi'_{n-1}(\lambda)\big|_{\lambda=1} = \frac{f'\left(t^*_{n-1}\right)}{f\left(t^*_{n-1}\right)}\left[F\left(t^*_{n-1}\right) - F\left(\hat{t}_{n-1}\right)\right] - (1-\alpha) f\left(t^*_{n-1}\right) + f\left(\hat{t}_{n-1}\right).$$

Hence,

$$\phi'_{n-1}(\lambda)\big|_{\lambda=0} - \phi'_{n-1}(\lambda)\big|_{\lambda=1} = \left[\frac{f'\left(\hat{t}_{n-1}\right)}{f\left(\hat{t}_{n-1}\right)} - \frac{f'\left(t^*_{n-1}\right)}{f\left(t^*_{n-1}\right)}\right]\left[F\left(t^*_{n-1}\right) - F\left(\hat{t}_{n-1}\right)\right].$$

The previous expression is strictly positive since F is increasing while $\frac{f'}{f}$ is decreasing. Therefore, $\phi'_{n-1}(\lambda)\big|_{\lambda=0} > \phi'_{n-1}(\lambda)\big|_{\lambda=1}$. Summing up: (i) $\phi_{n-1}(\cdot)$ is zero at $\lambda = 0$ and strictly positive at $\lambda = 1$; (ii) for $\lambda \in (0,1)$, $\phi'_{n-1}(\lambda)$ can change sign at most once; (iii) $\phi'_{n-1}(\lambda)$ is greater at $\lambda = 0$ than at $\lambda = 1$. Hence, $\phi_{n-1}(\lambda) > 0$ for any $\lambda \in (0,1]$. Thus, $\frac{\partial \bar{\omega}_n}{\partial p_{n-1}} > 0$.

Step 2: *$\bar{\omega}_n$ is increasing in $\bar{\omega}_{n-1}$*. Differentiating (A.57) with respect to $\bar{\omega}_{n-1}$ yields

$$\frac{\partial \bar{\omega}_n}{\partial \bar{\omega}_{n-1}} = \frac{(1-\lambda) f\left(\frac{p_{n-1}-\bar{\omega}_{n-1}}{1-\alpha}\right)}{f\left(F^{-1}\left(\lambda F\left(p_{n-1} - \overline{\omega}(s)\right) + (1-\lambda) F\left(\frac{p_{n-1}-\bar{\omega}_{n-1}}{1-\alpha}\right)\right)\right)}.$$

The expression above is clearly positive for any $\lambda \in (0,1]$. Hence, $\bar{\omega}_n$ is increasing in $\bar{\omega}_{n-1}$. ■

Proof of Proposition 7. Before proving the proposition, we characterize when a consumer buys in either period, and the type of the marginal buyer in period 2 under both rational inference and projection. In period 1, the quantity demanded is $D_1(p;\bar{\omega}(s)) = \lambda[1 - F(p - \bar{\omega}(s))] + (1-\lambda)[1 - F(p - \bar{\omega}_0)]$. Now consider what an agent with taste t who delays will infer from observing this quantity. They think that if informed agents expect a quality of $\hat{\omega}$, then the demand in period 1 is

$$\widehat{D}_1(p;\hat{\omega}|t) = \lambda\left[1 - F\left(\frac{p - \hat{\omega} - \alpha t}{1-\alpha}\right)\right] + (1-\lambda)\left[1 - F\left(\frac{p - \bar{\omega}_0 - \alpha t}{1-\alpha}\right)\right]. \quad \text{(A.60)}$$

Equating $D_1(p;\bar{\omega}(s))$ with $\widehat{D}_1(p;\hat{\omega}|t)$ allows us to solve for $\hat{\omega}_2(t)$, which denotes the perceived quality of an uninformed agent with taste t who delays. Assuming $T \sim U(\underline{t}, \bar{t})$, this solution is

$$\hat{\omega}_2(t) = \frac{\alpha}{\lambda}\bigg(p - (1-\lambda)\bar{\omega}_0 - t\bigg) + (1-\alpha)\bar{\omega}(s). \quad \text{(A.61)}$$

The marginal type in period 2 under projection is the $\hat{t}_2$ that solves $\hat{\omega}_2(\hat{t}_2) + \hat{t}_2 = p$, and hence

$$\hat{t}_2 = p - \left[\frac{\lambda(1-\alpha)}{\lambda - \alpha}\right]\bar{\omega}(s) + \left[\frac{\alpha(1-\lambda)}{\lambda-\alpha}\right]\bar{\omega}_0. \quad \text{(A.62)}$$

The marginal type in period 2 under rational inference is $t^*_2 = p - \bar{\omega}(s)$. Note that $\hat{t}_2 < t^*_2$ if and only if

$$p - \left[\frac{\lambda(1-\alpha)}{\lambda-\alpha}\right]\bar{\omega}(s) + \left[\frac{\alpha(1-\lambda)}{\lambda-\alpha}\right]\bar{\omega}_0 < p - \bar{\omega}(s) \quad \Leftrightarrow \quad \bar{\omega}(s) > \bar{\omega}_0. \tag{A.63}$$

Recall that the only types present in period 2 are those who did not buy in period 1; i.e., only those with $t \leq t_1^U \equiv p - \bar{\omega}_0$. Note that rational consumers in period 2 buy if and only if $t_2^* < t_2^U \Leftrightarrow \bar{\omega}(s) > \bar{\omega}_0$. Condition (A.63) thus implies that the same is true under projection: $\hat{t}_2 < t_2^U \Leftrightarrow \bar{\omega}(s) > \bar{\omega}_0$; hence, projectors in period 2 only buy when the quality is higher than expected.

Part 1. Suppose $\bar{\omega}(s) > \bar{\omega}_0$. Under rational inference, the interval of types who buy in period 2 is $[t_2^*, t_1^U]$. Under projection, this interval is $[\hat{t}_2, t_1^U]$, where $\hat{t}_2 < t_2^*$ by (A.63). Hence, the quantity demanded in period 2 under projection exceeds the rational benchmark. Moreover, using the expressions above for $\hat{t}_2$ and t_2^*, the interval of types who wrongly adopt the good is $t_2^* - \hat{t}_2 = (\bar{\omega}(s) - \bar{\omega}_0)(\alpha(1-\lambda))/(\lambda-\alpha)$. The measure of this interval is clearly increasing in α and in $\bar{\omega}(s) - \bar{\omega}_0$.

Now consider the set of types who buy in period 2 yet hold a quality expectation exceeding the rational expectation, $\mathcal{T}_O(s) \equiv \{t \in [\hat{t}, t_1^U] \mid \tilde{\omega}_2(t) > \bar{\omega}(s)\}$. This set represents the buyers who overestimate quality and will, on average, be disappointed by buying ex post; that is, $t \in \mathcal{T}_O(s) \Rightarrow \mathbb{E}[\omega - \hat{\omega}(t)|s] < 0$. Let $\tilde{t}$ be the type in period 2 who infers correctly; i.e., $\hat{\omega}_2(\tilde{t}) = \bar{\omega}(s)$. From (A.61), we have $\tilde{t} = p - \lambda\bar{\omega}(s) - (1-\lambda)\bar{\omega}_0$. Since $\hat{\omega}_2(t)$ is decreasing in t, $\hat{\omega}_2(t) > \bar{\omega}(s)$ for all $t < \tilde{t}$ and hence $\mathcal{T}_O(s) = [\hat{t}, \tilde{t})$. Since $\bar{\omega}(s) > \bar{\omega}_0$, we have $\tilde{t} \in (t_2^*, t_1^U)$ given that $\lambda \in (0,1)$. In contrast to rational learning, $\tilde{t} > t_2^*$ implies that some projecting buyers who correctly adopt the good (i.e., their expected valuation exceeds the price) will systematically experience disappointment, on average.

Part 2. Suppose $\bar{\omega}(s) < \bar{\omega}_0$. As discussed prior to Part 1, $\bar{\omega}(s) < \bar{\omega}_0$ implies that no consumers buy in period 2 under rational inference or under projection. Hence, outcomes in this case match the rational benchmark. ■

Proof of Proposition 8. Before proving the proposition, we derive some preliminary results on the nature of uninformed agents' biased inference rules and the equilibrium quantity demanded.

Let $t^* \equiv p - \bar{\omega}(s)$ be the marginal informed type (i.e., an informed type strictly prefers to buy a positive quantity if and only if $t > t^*$). The aggregate demand of informed agents is then

$$D^I(p; \bar{\omega}(s)) = \int_{t^*}^{\bar{t}} x^*(p; \bar{\omega}(s), t)dF(t) = -[1 - F(t^*)]t^* + \int_{t^*}^{\bar{t}} \tilde{t}f(\tilde{t})d\tilde{t} \tag{A.64}$$

since $x^*(p; \bar{\omega}(s), t) = \bar{\omega}(s) - p + t$. Let $H(t) \equiv -[1 - F(t)]t + \int_{\tilde{t} \geq t} \tilde{t}f(d)d\tilde{t}$. Now consider the demand function among agents with a quality expectation of $\hat{\omega}$ from the perspective of an uninformed agent with taste t. This agent believes the marginal type is $\hat{t} = p - \hat{\omega}$, and hence perceives

$$\begin{aligned}
\widehat{D}^I(p; \hat{\omega}|t) &= -[1 - \widehat{F}(\hat{t}|t)]\hat{t} + \int_{\hat{t}}^{\bar{t}(t)} \tilde{t}\hat{f}(\tilde{t}|t)d\tilde{t} \\
&= -\left[1 - F\left(\frac{\hat{t} - \alpha t}{1-\alpha}\right)\right]\hat{t} + \int_{\hat{t}}^{\bar{t}(t)} \tilde{t}\frac{1}{1-\alpha}f\left(\frac{\tilde{t} - \alpha t}{1-\alpha}\right)d\tilde{t}.
\end{aligned} \tag{A.65}$$

Consider a change of variables with $x = \frac{\tilde{t} - \alpha t}{1-\alpha}$. Recalling that $\bar{t}(t) = \alpha t + (1-\alpha)\bar{t}$, expression

(A.65) can be re-written as

$$
\begin{aligned}
\widehat{D}^I(p;\hat{\omega}|t) &= -\left[1-F\left(\frac{\hat{t}-\alpha t}{1-\alpha}\right)\right]\hat{t}+\int_{\frac{\hat{t}-\alpha t}{1-\alpha}}^{\bar{t}}[\alpha t+(1-\alpha)x]f(x)\,dx \\
&= -\left[1-F\left(\frac{\hat{t}-\alpha t}{1-\alpha}\right)\right][\hat{t}-\alpha t]+(1-\alpha)\int_{\frac{\hat{t}-\alpha t}{1-\alpha}}^{\bar{t}} xf(x)\,dx \\
&= (1-\alpha)H\left(\frac{\hat{t}-\alpha t}{1-\alpha}\right), \qquad \text{(A.66)}
\end{aligned}
$$

where H is defined below (A.64).

An uninformed projecting agent's inference rule, $\hat{\omega}(d|t)$, is obtained by finding the perceived marginal type $\hat{t}(d|t)$ that solves $\widehat{D}^I(p;\hat{\omega}|t)=(1-\alpha)H\left(\frac{\hat{t}-\alpha t}{1-\alpha}\right)=d$, and then setting $\hat{\omega}(d|t)=p-\hat{t}$. We now use the Implicit Function Theorem to show that a projector's biased inference rule is linearly decreasing in t with slope α.

Let $L(x;d)=(1-\alpha)H(x)-d$. Note that an agent infers a marginal type $\hat{t}(d|t)$ equal to the value of $\hat{t}$ that solves $L\left(\frac{\hat{t}-\alpha t}{1-\alpha};d\right)=0$. Thus,

$$
\begin{aligned}
\frac{\partial\hat{t}(d|t)}{\partial t} &= -\left(\frac{\partial}{\partial t}L\left(\frac{\hat{t}-\alpha t}{1-\alpha};d\right)\right)\left(\frac{\partial}{\partial\hat{t}}L\left(\frac{\hat{t}-\alpha t}{1-\alpha};d\right)\right)^{-1}\Bigg|_{\hat{t}=\hat{t}(d|t)} \\
&= -\left(-\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{1-\alpha}\right)^{-1}\Bigg|_{\hat{t}=\hat{t}(d|t)}=\alpha.
\end{aligned}
$$

Since $\hat{\omega}(d|t)=p-\hat{t}(d|t)$, $\frac{\partial}{\partial t}\hat{\omega}(d|t)=-\alpha$. Thus, we can write any uninformed type's inferred value of $\bar{\omega}(s)$ upon observing aggregate demand as $\hat{\omega}(d|t)=\tilde{\omega}(d)-\alpha t$, where $\tilde{\omega}(d)$ is independent of t. While we will not explicitly solve for $\tilde{\omega}(d)$ (which will depend on F and α), we now argue that, in equilibrium, the aggregate quantity demanded by uninformed agents is equal to the aggregate quantity demanded by informed agents. To see this, we first derive the aggregate quantity demanded by uninformed agents. Since $\hat{\omega}(d|t)=\tilde{\omega}(d)-\alpha t$, an uninformed type t will demand $\tilde{\omega}(d)-p+(1-\alpha)t$ units. Thus, the truly marginal type among uninformed agents is $\hat{t}=(p-\tilde{\omega}(d))/(1-\alpha)$, and the aggregate demand among uninformed types is

$$
\begin{aligned}
D^U(p;\tilde{\omega}(d)) &= \int_{\hat{t}=\frac{p-\tilde{\omega}(d)}{1-\alpha}}^{\bar{t}}[\tilde{\omega}(d)-p+(1-\alpha)t]dF(t) \\
&= (1-\alpha)\int_{\hat{t}=\frac{p-\tilde{\omega}(d)}{1-\alpha}}^{\bar{t}}\left[-\frac{p-\tilde{\omega}(d)}{1-\alpha}+t\right]dF(t)=(1-\alpha)H\left(\frac{p-\tilde{\omega}(d)}{1-\alpha}\right). \qquad \text{(A.67)}
\end{aligned}
$$

Note that $\frac{\partial}{\partial d}D^U(p;\tilde{\omega}(d))=-H\left(\frac{p-\tilde{\omega}(d)}{1-\alpha}\right)\frac{\partial\tilde{\omega}(d)}{\partial d}$, and $\frac{\partial\tilde{\omega}(d)}{\partial d}=\frac{\partial\hat{\omega}(d|t)}{\partial d}$; hence, $\hat{t}(d|t)=p-\hat{\omega}(d|t)$

implies $\frac{\partial \tilde{\omega}(d)}{\partial d} = -\frac{\partial \hat{t}(d|t)}{\partial d}$. Since $\hat{t}(d|t)$ solves $L\left(\frac{\hat{t}-\alpha t}{1-\alpha}; d\right) = 0$, we have

$$\begin{aligned}
\frac{\partial \hat{t}(d|t)}{\partial d} &= -\left(\frac{\partial}{\partial d} L\left(\frac{\hat{t}-\alpha t}{1-\alpha}; d\right)\right)\left(\frac{\partial}{\partial \hat{t}} L\left(\frac{\hat{t}-\alpha t}{1-\alpha}; d\right)\right)^{-1}\Bigg|_{\hat{t}=\hat{t}(d|t)} \\
&= -(-1)\left((1-\alpha)H\left(\frac{\hat{t}-\alpha t}{1-\alpha}\right)\frac{1}{1-\alpha}\right)^{-1}\Bigg|_{\hat{t}=\hat{t}(d|t)} \\
&= \left(H\left(\frac{\hat{t}-\alpha t}{1-\alpha}\right)\right)^{-1}\Bigg|_{\hat{t}=\hat{t}(d|t)} = \left(H\left(\frac{\hat{t}(d|t)-\alpha t}{1-\alpha}\right)\right)^{-1} \\
&= \left(H\left(\frac{p-\hat{\omega}(d|t)-\alpha t}{1-\alpha}\right)\right)^{-1} = \left(H\left(\frac{p-\tilde{\omega}(d)}{1-\alpha}\right)\right)^{-1}, \qquad \text{(A.68)}
\end{aligned}$$

and therefore

$$\frac{\partial \tilde{\omega}(d)}{\partial d} = -\frac{\partial \hat{t}(d|t)}{\partial d} \quad \Rightarrow \quad \frac{\partial \tilde{\omega}(d)}{\partial d} = -\left(H\left(\frac{p-\tilde{\omega}(d)}{1-\alpha}\right)\right)^{-1}, \qquad \text{(A.69)}$$

which further implies that $\frac{\partial}{\partial d} D^U(p; \tilde{\omega}(d)) = 1$. Thus, D^U as a function of the observed equilibrium quantity must vary identically with d; that is, $D^U(p; \tilde{\omega}(d)) = d + c$ for some constant c. But the only constant generically consistent with the required equilibrium condition of $d = \lambda D^I(p; \bar{\omega}(s)) + (1-\lambda)D^U(p; \tilde{\omega}(d))$ is $c = 0$. Thus, in equilibrium, $\tilde{\omega}(d)$ must be such that $D^U(p; \tilde{\omega}(d)) = D^I(p; \bar{\omega}(s))$, so that $d = D^I(p; \bar{\omega}(s))$. For shorthand, let $\hat{\omega}(t)$ denote $\hat{\omega}(d|t)$ evaluated at $d = D^I(p; \bar{\omega}(s))$.

Part 1. As previously established, an uninformed agent with taste t forms an estimate of ω equal to $\hat{\omega}(t) = \tilde{\omega}(d) - \alpha t$, where $\tilde{\omega}(d)$ is independent of t. Thus, $\hat{\omega}(t)$ is decreasing in t whenever $\alpha > 0$.

Part 3. We prove Part 3 before Part 2. As argued above, in equilibrium $D^U(p; \tilde{\omega}(d)) = D^I(p; \bar{\omega}(s))$ must hold. Recall that $t^* = p - \bar{\omega}(s)$ and $\hat{t} = (p - \tilde{\omega}(d))/(1-\alpha)$ are the marginal informed and uninformed types, respectively. From (A.64) and (A.67), we have $D^I(p; \bar{\omega}(s)) = H(t^*)$ and $D^U(p; \tilde{\omega}(d)) = (1-\alpha)H(\hat{t})$. Hence, in equilibrium, we must have $H(t^*) = (1-\alpha)H(\hat{t})$. Since H is strictly decreasing, $\hat{t} < t^*$ whenever $\alpha > 0$.

Part 2. Next, we argue that the uninformed marginal type overestimates ω; that is $\hat{t} < t^*$ if and only if $(p - \tilde{\omega}(d))/(1-\alpha) < p - \bar{\omega}(s)$ which is equivalent to $\tilde{\omega}(d) > (1-\alpha)\bar{\omega}(s) + \alpha p$. This in turn implies that $\hat{\omega}(\hat{t}) > \bar{\omega}(s)$ since $\hat{\omega}(\hat{t}) = \tilde{\omega}(d) - \alpha\hat{t} = \tilde{\omega}(d) - \alpha(p - \tilde{\omega}(d))/(1-\alpha)$. Thus, $\hat{\omega}(\hat{t}) > \bar{\omega}(s)$. Furthermore, there must exist a $\tilde{t} \in (\hat{t}, \bar{t})$ such that $\hat{\omega}(\tilde{t}) = \bar{\omega}(s)$. If such a type did not exist, then the fact that $\hat{\omega}(t) = \tilde{\omega}(d) - \alpha t$ would imply that all uninformed types who buy in equilibrium overestimate $\bar{\omega}(s)$. But this, together with the fact that $\hat{t} < t^*$, would then imply that $D^U(p; \tilde{\omega}(d)) > D^I(p; \bar{\omega}(s))$ since, relative to informed types, a wider interval of uninformed types buy and they all overestimate $\bar{\omega}(s)$. Yet this contradicts the requirement that $D^U(p; \tilde{\omega}(d)) = D^I(p; \bar{\omega}(s))$, and hence there exists a $\tilde{t} \in (\hat{t}, \bar{t})$ such that $\hat{\omega}(\tilde{t}) = \bar{\omega}(s)$; moreover, $\hat{\omega}(t) = \tilde{\omega}(d) - \alpha t$ implies that $\hat{\omega}(t) > \bar{\omega}(s)$ for $t < \tilde{t}$ and $\hat{\omega}(t) < \bar{\omega}(s)$ for $t > \tilde{t}$. Since an uninformed type demands $x^*(p; \hat{\omega}(t), t) = \hat{\omega}(t) + t - p$, we additionally have $x^*(p; \hat{\omega}(t), t) > x^*(p; \bar{\omega}(s), t)$ for $t < \tilde{t}$ and $x^*(p; \hat{\omega}(t), t) < x^*(p; \bar{\omega}(s), t)$ for $t > \tilde{t}$.

Part 4. Note that $\left|x^*(p; \hat{\omega}(t), t) - x^*(p; \bar{\omega}(s), t)\right| = \left|\hat{\omega}(t) - \bar{\omega}(s)\right| = \left|\tilde{\omega}(d) - \bar{\omega}(s) - \alpha t\right|$. By definition of $\tilde{t}$, $\hat{\omega}(\tilde{t}) = \tilde{\omega}(d) - \alpha\tilde{t} = \bar{\omega}(s)$. Thus, $\left|\tilde{\omega}(d) - \bar{\omega}(s) - \alpha t\right| = \left|\tilde{\omega}(d) - [\tilde{\omega}(d) - \alpha\tilde{t}] - \alpha t\right| = \left|\alpha\tilde{t} - \alpha t\right|$, and hence $\left|x^*(p; \hat{\omega}(t), t) - x^*(p; \bar{\omega}(s), t)\right| = \alpha|t - \tilde{t}|$. ■